Spontaneous parity violation in extreme conditions: an effective lagrangian analysis

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We investigate how large baryon densities (and possibly high temperatures) may induce spontaneous parity violation in the meson sector of QCD. The analysis at intermediate energy scales is done by using an extended σ-model lagrangian that includes two scalar and two pseudoscalar multiplets and fulfills low-energy QCD constraints. We elaborate on a novel mechanism of parity breaking previously proposed by the authors based on the interplay between lightest and heavier meson condensates, which therefore cannot be realized in the simplest σ model. We emphasize that the mechanism proposed here differs from the old idea of pion condensation advocated originally by Migdal. The results are relevant for an idealized homogeneous and infinite nuclear (quark) matter where the influence of density can be examined with the help of a constant chemical potential. The model is able to describe satisfactorily the first-order phase transition to stable nuclear matter, and predicts a second-order phase transition to a state where parity is spontaneously broken. We argue that the parity breaking phenomenon is quite generic when a large enough chemical potential is present. Current quark masses are explicitly taken into account in this work and shown not to change the general conclusions. We expect that our approach will be adequate for dense nuclear matter of a few normal densities where quark percolation does not yet play a significant role.

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I. INTRODUCTION

Emergent parity violation for sufficiently large values of the baryon chemical potential (and/or temperature) has been attracting much interest during several decades (see reviews [1]). It is actively looked for in both in dense hadron matter (in heavy ion collisions at intermediate energies and neutron stars) and in strongly interacting quark-gluon matter ("quark-gluon plasma" in heavy ion collisions at very high energies and in quark stars). At finite baryon density pion condensation is a plausible possibility (conjectured in nuclear physics long ago in [2]. Although an attractive idea, the consensus reached by workers in the field is that it is not possible in simple models of pion-nucleus interaction. However in this work this idea will be vindicated and in fact claimed to be a very realistic possibility in more elaborated models.

More recently the phenomenon of parity breaking was assumed to be present in meta-stable nuclear bubbles with non-zero axial charge created in hot nuclear matter [3] and/or in the presence of a strong background magnetic fields [4, 5]. It was also shown [6] that the associated axial chemical potential causes the distortion of energy spectrum of photons and vector particles (ρ and ω mesons) due to Chern-Simons interaction and as a consequence of parity breaking a remarkable mass splitting of their left- and right-handed polarizations appears. This phenomenon is however theoretically somewhat different in its origin and unrelated to the present paper, which is devoted to search for phase transitions with generation of neutral pseudoscalar isospin-one condensate.

Parity violation may in fact accompany the transitions to open color phases like the color-flavor locking (CFL) [7] or superconducting ones [8], but these regions are beyond the scope of the present paper.

Some time ago it was proved in [9] that parity and vector flavor symmetry could not undergo spontaneous symmetry breaking in a vector like theory such as QCD at normal vacuum conditions at zero chemical potential [10]. Finite baryon density however may result in a breaking of parity invariance by simply

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circumventing the hypothesis of the theorem. Indeed the presence of a finite chemical potential leads to the appearance of a constant imaginary zeroth-component of a vector field and the conditions under which the results of [9] were proven are not fulfilled anymore.

Can this possibility of parity breaking be realized in nature and what would then be its experimental manifestation? In order to answer the first question we might appeal to lattice QCD for help and in fact this possibility has been studied intensively for quite some time. However, finite density simulations are notoriously difficult (the fermion determinant is in general complex for a non-zero chemical potential μ) and one has to resort to more involved techniques such as determining the phase of the determinant separately, Taylor expansions in μ or analytic continuation to imaginary chemical potential. On top of that Pauli blocking is at work for large values of μ, etc. For a comprehensive review of the ongoing issues, see for instance [13–15]. Thus the lattice results for sufficiently large values of the baryon chemical potential (where the effect is expected to appear) are not known rigorously yet [16] .

In this work we shall attempt to explore the interesting issue of parity breaking employing effective lagrangian techniques, useful to explore the range of nuclear densities where the hadron phase still persists and quark percolation does not occur yet. Our effective lagrangian is a realization of the generalized linear σ model, but including the two lowest lying resonances in each channel, those that are expected to play a role in this issue. This seems to be the minimal model where the interesting possibility of parity breaking can be realized. The use of effective lagrangians is also crucial to answer the second question of interest, namely how would parity breaking originating from a finite baryon density eventually reflect in hadronic physics. This work is an extension of the preliminary results presented in [18, 19].

An oldest, pre-QCD attempt to give the lagrangian description for two multiplets of scalar and pseudoscalar mesons was done in [20] with a reduced set of operators and a chiral symmetry breaking (CSB) pattern not quite compatible with QCD. In turn, we have been rather inspired by our previous works on extended quark models [21–23] where two different schemes with linear and non-linear realization of chiral symmetry were adopted to incorporate heavy pions and scalar mesons within an effective quark model with quark self-interactions. A certain resemblance can be also found with the model [24] where two $SU(3)_F$ multiplets have been associated with two-quark and four-quark meson states although we do not share the assumption in [24] concerning the dominance of four-quark component in radially excited mesons. The model in [25] is also of relevance in studying of vacuum for extended σ models.

The paper is organized as follows. In section 2 the bosonization of QCD quark currents in the color-singlet sector is discussed and the ingredients of the generalized σ model are indicated. Some subtleties associated to the choice of the low-energy effective hadronic theory are identified. Too simple models are not rich enough to explore all the different phases that the presence of parity violation opens for us. In section 3 we introduce the σ model with two multiplets of isosinglet scalar and isotriplet pseudoscalar fields. The effective potential for two multiplets of scalar and pseudoscalar mesons is obtained and the mass-gap equations and second variations at the minima are derived. We impose on the model the conditions for it not to lead to spontaneous breaking of parity for vanishing baryon chemical potential. This condition bounds a combination of several coupling constants. In section 4 the freedom of reparameterization is exploited to reduce the number of relevant constants so that the mass-gap equations allow leads to physically inequivalent solutions.

In section V we shall introduce the finite chemical potential and temperature and see how they modify the effective theory and the vacuum state. Temperature and baryon chemical potential appear through the one-quark loop free energy; the only one that needs consideration in the large N_c limit as the contribution of meson loops is neglected in the mean field approach. Different thermodynamic properties of the model are obtained for non-zero temperature and chemical potential (pressure, energy density and entropy).

In the subsequent sections we ignore temperature effects and concentrate on the finite density case. In order to describe adequately the saturation point transition to stable nuclear matter we supplement the effective lagrangian with a ω meson coupling to the isosinglet quark current. It is known for some time [26] that ω mesons contribute to the repulsive part of nuclear forces and thereby they support the formation of stable nuclear matter. Indeed, the correct first-order phase transition at the saturation point is found and we prove that the chiral collapse affecting the simplest σ models and/or the one-mutiplet Nambu-Jona-Lasinio (NJL) models does not occur here. Once the saturation point is established one can define (in)compressibilities and perform the matching between quarks and nuclear matter.

In section 6 we investigate the emergence of spontaneous parity breaking (SPB) phase. The mass gap equations and critical lines for the parity breaking phase transition are derived and corrections beyond the chiral limit taken into account to the leading order in quark masses. An interesting question examined is on how the saturation point meets spontaneous parity breaking.

In section 7 we study the approach to the SPB phase transition both in the chiral limit and beyond. It

is established that this phase transition is of second order. The second variation of the effective potential leads to the mass spectrum for scalar/pseudoscalar states in this phase. We remind that in the SPB phase strictly speaking there are no genuine scalar or pseudoscalar states as each of massive states can equally well decay into two and three scalars. In section 8 the kinetic terms are considered in order to determine the physical masses of mesons. In the SPB phase the masses of the lightest states are calculated beyond the chiral limit and the existence of two massless Goldstone bosons verified. We summarize our findings and propose possible signals of the SPB phase in the conclusions section.

The range of intermediate nuclear densities where our effective lagrangian is applicable is of high interest as they may be reached in both compact stars [27] and heavy-ion collisions [28]. Its relevance can be qualitatively motivated by the fact that at substantially larger densities typical distances between baryons are shrinking considerably and meson excitations with Compton wave lengths much shorter than a pion one start playing an important role.

There are previous studies dealing with the problem of strong interactions at zero temperature and finite chemical potential: depending on a value of nuclear density, a variety of methods are involved from using meson-nucleon [1, 29] or quark-meson[27, 30] lagrangians for low-dense nuclear matter to models of the NJL type[31] for high-dense quark matter[8]. At very large chemical potentials, when diquark condensation gives rise to a new CFL phase [8], one can guess[7] an effective lagrangian describing the lightest degrees of freedom and the conditions for SPB due to pion or kaon condensation.

Recent papers are more concerned with case of non-zero chemical isospin potential[32] (where comparison with lattice results is possible). In our view these works are somewhat incomplete and hence they may not lead to conclusive answers for intermediate nuclear densities when quark percolation does not yet occur but pion-nucleon theory is certainly insufficient to give an adequate description. Although the issue of SPB in hadronic phase has been touched upon in the pion-nucleon theory[1, 29, 33] and in NJL models [34] the validity of the models used is not quite clear for intermediate nuclear densities. The reason is discussed in the next section: they are not rich enough to explore the subtle phenomenology involved. There are also promising recent attempts [35] to describe the large baryon density influence on hadron properties.

II. BOSONIZATION OF QCD IN THE COLOR-SINGLET SECTOR

In order to elaborate an effective lagrangian for meson states starting from QCD we revisit the properties of color-singlet (quasi)local quark currents in the QCD vacuum with spontaneously broken chiral symmetry. This phenomenon emerges due to a non-zero value of quark condensate $\langle \bar{q}q \rangle$ and can be associated to the CSB scale $\Lambda_{CSB} \sim 1GeV (\equiv \Lambda$ for brevity). This CSB due to quark condensation makes the quark bilinears to be interpolating operators for meson fields (in the limit of large colors). In particular, a scalar/pseudoscalar quark density effectively describes the creation or annihilation of scalar/pseudoscalar mesons

$$\bar{q}q(x) \simeq \Lambda^2 \sum_l Z_l^{(1)} \Sigma_l(x); \qquad \bar{q}\gamma_5 \tau^i q(x) \simeq \Lambda^2 \sum_l Z_l^{(1)} \Pi_l^i(x), \tag{1}$$

with the normalized meson fields Σ_l, Π_l^i describing the families of resonances with the same quantum numbers but increasing masses (radial Regge trajectories)[36] and the set of normalization constants $Z_l^{(1)}$. The constituent quark fields are denoted as $\bar{q}, q$ and τ^i stand for Pauli matrices. At this stage we consider the chiral limit of zero current quark masses. Accordingly the global chiral covariance of quark operators (1) is transmitted to the set of boson operators leading to an equal normalization of scalar and pseudoscalar fields. This is a basic framework of linear sigma models [37] (see next section). In this paper we restrict ourselves with consideration of two light flavors related to u, d quarks and therefore the approximate chiral symmetry of the quark sector is $SU(2)_L \times SU(2)_R$.

Keeping in mind confinement we have retained in (1) only the one-resonance states as leading ones while being aware of that the total saturation of quark currents includes, of course, also multi-resonance states. Thus we use the large-N_c approach where resonances behave like true elementary particles with zero widths and multi-resonant states can be neglected.

Let us comment a bit more on the previous relation. On the left-hand side one sees an operator of canonical dimension 3 whereas on the right-hand side one finds field operators of canonical dimension 1. This drastic change in dimensions is a consequence of CSB and it modifies the dimensional analysis of what must be included into an effective lagrangian. More exactly, in order to replace the non-perturbative regime of QCD at low and intermediate energies by a hadron effective lagrangian one has to apply this dimensional counting

in the CSB phase [22] to all possible combinations of color-singlet operators arising in the chiral expansion in inverse powers of the CSB scale Λ.

To be specific the chiral invariant local operators playing the leading role in the low-energy effective lagrangian for meson dynamics are

$$\frac{1}{\Lambda^2}\left[(\bar{q}q)^2 - \bar{q}\gamma_5\tau_i q\bar{q}\gamma_5\tau^i q\right] \simeq \Lambda^2 \sum_{l,m} Z^{(2)}_{lm}[\Sigma_l\Sigma_m + \Pi^i_l\Pi_{i,m}] + \mathcal{O}(1);$$
$$\frac{1}{\Lambda^8}[(\bar{q}q)^2 - \bar{q}\gamma_5\tau_i q\bar{q}\gamma_5\tau^i q]^2 \simeq \sum_{l,m,n,r} Z^{(4)}_{lmnr}[\Sigma_l\Sigma_m + \Pi^i_l\Pi_{i,m}][(\Sigma_n\Sigma_r + \Pi^i_n\Pi_{i,r}] + \mathcal{O}(\frac{1}{\Lambda^2});$$
$$\frac{1}{\Lambda^4}[\partial_\mu(\bar{q}q)]^2 - \partial_\mu(\bar{q}\gamma_5\tau_i q)\partial^\mu(\bar{q}\gamma_5\tau^i q)] \simeq \sum_{l,m} \tilde{Z}^{(2)}_{lm}[\partial_\mu\Sigma_l\partial^\mu\Sigma_m + \partial_\mu\Pi^i_l\partial^\mu\Pi_{i,m}] + \mathcal{O}(\frac{1}{\Lambda^2}), \tag{2}$$

where the matrices $Z^{(2)}_{lm}, Z^{(4)}_{lmnr}, \tilde{Z}^{(2)}_{lm}$ must be symmetric under transposition of indices in order to provide global chiral invariance. The terms quadratic in scalar fields trigger (if their coefficient is of the right sign an instability that leads to CSB in the effective meson theory (see. e.g. [38]).

The above set of operators is not complete and can be extended with the help of form factors that are polynomials in derivatives [22]. For example, using the same CSB scale Λ one can add into the effective quark lagrangian the vertices built of the elements

$$\bar{q}\frac{\overleftrightarrow{\partial}^{2k}}{\Lambda^{2k}}q(x) \simeq \Lambda^2 \sum_l Z^{(1),k}_l \Sigma_l(x); \qquad \bar{q}\gamma_5\tau^i\frac{\overleftrightarrow{\partial}^{2k}}{\Lambda^{2k}}q(x) \simeq \Lambda^2 \sum_l Z^{(1),k}_l \Pi^i_l(x), \tag{3}$$

which may give numerically comparable contributions for several k [22].

III. GENERALIZED SIGMA-MODEL

A. Effective potential for two multiplets

The simplest hadronic effective theory is the linear sigma-model of Gell-Mann and Levy[37], which contains a multiplet of the lightest scalar σ and pseudoscalar π^a fields. Spontaneous CSB emerges due to a non-zero value for $\langle\sigma\rangle \sim \langle\bar{q}q\rangle/F_\pi^2$. Current algebra techniques indicate that in order to relate this model to QCD one has to choose a real condensate for the scalar density, with its sign opposite to current quark masses, and avoid any parity breaking due to a v.e.v. of the pseudoscalar density. The introduction of a chemical potential does not change the phase of the condensate and therefore does not generate any parity breaking. This is just fine because in normal conditions parity breaking does not take place in QCD. However, if two different scalar fields condense with a relative phase between the two v.e.v.'s the opportunity of spontaneous parity breaking may arise.

Let us consider a model with two multiplets of scalar ($\tilde{\sigma}_j$) and pseudoscalar ($\tilde{\pi}^a_j$) fields

$$H_j = \tilde{\sigma}_j\mathbf{I} + i\hat{\pi}_j, \quad j = 1, 2; \quad H_j H_j^\dagger = (\tilde{\sigma}_j^2 + (\tilde{\pi}^a_j)^2)\mathbf{I}, \tag{4}$$

with $\hat{\pi}_j \equiv \tilde{\pi}^a_j\tau^a$ with τ^a being a set of Pauli matrices. We shall deal with a scalar system globally symmetric respect to $SU(2)_L \times SU(2)_R$ rotations in the exact chiral limit and next consider the soft breaking of chiral symmetry by current quark masses. We should think of these two chiral multiplets as representing the two lowest-lying radial states for a given J^{PC}. Of course one could add more multiplets, representing higher radial and spin excitations, to obtain a better description of QCD, but the present model, without being completely realistic, already possesses all the necessary ingredients to study SPB. Inclusion of higher-mass states would be required at substantially larger densities when typical distances between baryons are shrinking considerably and meson excitations with Compton wave lengths much shorter than a pion one start playing an important role.

Let us define the effective potential of this generalized sigma-model. First we write the most general

Hermitian potential at zero μ,

$$V_{\text{eff}} = \frac{1}{2}\text{tr}\Bigg\{-\sum_{j,k=1}^{2} H_j^\dagger \Delta_{jk} H_k + \lambda_1 (H_1^\dagger H_1)^2 + \lambda_2 (H_2^\dagger H_2)^2 + \lambda_3 H_1^\dagger H_1 H_2^\dagger H_2$$
$$+\frac{1}{2}\lambda_4 (H_1^\dagger H_2 H_1^\dagger H_2 + H_2^\dagger H_1 H_2^\dagger H_1) + \frac{1}{2}\lambda_5 (H_1^\dagger H_2 + H_2^\dagger H_1) H_1^\dagger H_1$$
$$+\frac{1}{2}\lambda_6 (H_1^\dagger H_2 + H_2^\dagger H_1) H_2^\dagger H_2 \Bigg\} + \mathcal{O}(\frac{|H|^6}{\Lambda^2}), \tag{5}$$

which contains 9 real constants $\Delta_{jk}, \lambda_A;\ A = 1, \ldots, 6$. However this set of constants can be reduced (see sect.IV). QCD bosonization rules in the large N_c limit prescribe $\Delta_{jk} \sim \lambda_A \sim N_c$. The neglected terms will be suppressed by inverse power of the CSB scale $\Lambda \sim 1$ GeV. If we assume the v.e.v. of H_j to be of the order of the constituent mass $0.2 \div 0.3$ GeV, it is reasonable to neglect these terms.

One could add five more terms (breaking parity manifestly): an imaginary part of Δ_{12} with an operator

$$i\text{tr}\left\{(H_1^\dagger H_2 - H_2^\dagger H_1)\right\}, \tag{6}$$

and three more operators

$$i\text{tr}\left\{H_1^\dagger H_2 H_1^\dagger H_2 - H_2^\dagger H_1 H_2^\dagger H_1\right\}, \quad i\text{tr}\left\{(H_1^\dagger H_2 - H_2^\dagger H_1) H_1^\dagger H_1\right\},$$
$$i\text{tr}\left\{(H_1^\dagger H_2 - H_2^\dagger H_1) H_2^\dagger H_2\right\}. \tag{7}$$

As well as an operator with disconnected trace seems to complete the full set of operators

$$\left(\text{tr}\left\{H_1^\dagger H_2\right\}\right)^2 + \left(\text{tr}\left\{H_1 H_2^\dagger\right\}\right)^2. \tag{8}$$

However after specifying the v.e.v. $\langle H_1 \rangle = \langle \sigma_1 \rangle$ one can use the global invariance of the model to factor out the Goldstone boson fields with the help of the chiral parameterization

$$H_1(x) = \sigma_1(x)\xi^2(x) = \sigma_1(x)\exp\left(i\frac{\pi_1^a \tau_a}{F_0}\right); \quad H_2(x) = \xi(x)\Big(\sigma_2(x) + i\hat{\pi}_2(x)\Big)\xi(x), \tag{9}$$

which differs from eq. (4) in notation. This kind of parameterization preserves the parities of $\sigma_2(x)$ and $\hat{\pi}_2$ to be even and odd respectively in the absence of SPB. Then the contribution of the four additional operators (6),(7) vanishes identically, whereas the operator (8) turns out to be a combination of operators with constants $\lambda_{3,4}$. Finally the potential (5) is further simplified to

$$V_{\text{eff}} = -\sum_{j,k=1}^{2} \sigma_j \Delta_{jk} \sigma_k - \Delta_{22}(\pi_2^a)^2 + \lambda_2\Big((\pi_2^a)^2\Big)^2 + (\pi_2^a)^2\Big((\lambda_3 - \lambda_4)\sigma_1^2 + \lambda_6 \sigma_1 \sigma_2 + 2\lambda_2 \sigma_2^2\Big)$$
$$+\lambda_1 \sigma_1^4 + \lambda_2 \sigma_2^4 + (\lambda_3 + \lambda_4)\sigma_1^2 \sigma_2^2 + \lambda_5 \sigma_1^3 \sigma_2 + \lambda_6 \sigma_1 \sigma_2^3. \tag{10}$$

The current quark mass m_q corresponds to the average of the external scalar sources $M_j(x) = s_j(x) + i\tau^a p_j^a(x)$, namely, $\langle M_j(x)\rangle = -\frac{1}{2}d_j m_q$ and thus the relevant new terms beyond the chiral limit can be produced with the help of the formal replacement $H_j \to c_j m_q$ in all quadratic and quartic operators included in (5) and by adding these new terms with new constants into the effective potential. We will consider the two flavor case and retain the softest terms linear in H_j and m_q, thereby neglecting terms cubic in scalar fields exploiting the non-linear equivalence transformation $H_j \to \sum_{k,l,m=1,2} b_{jklm} H_k H_l^\dagger H_m$. With an appropriate choice of external scalar sources only one type of Hermitian structures is possible, $\sum_{j=1,2}$ tr $(M_j^\dagger H_j + H_j^\dagger M_j)$, i.e. we add two new terms to our effective potential (10)

$$-\frac{1}{2}m_q \text{tr}\left[d_1(H_1 + H_1^\dagger) + d_2(H_2 + H_2^\dagger)\right]. \tag{11}$$

Making use of our chiral parametrization of the fields H_j through the chiral field

$$U \equiv \xi^2 = \cos\frac{|\pi_1^a|}{F_0} + i\frac{\tau^a \pi_1^a}{|\pi_1^a|}\sin\frac{|\pi_1^a|}{F_0}, \tag{12}$$

one derives the following extension of the effective potential (10)

$$\Delta V_{\text{eff}}\,(m_q) = 2m_q \left[-(d_1\sigma_1 + d_2\sigma_2)\cos\frac{|\pi_1^a|}{F_0} + d_2 \frac{\pi_1^a \pi_2^a}{|\pi_1^a|}\sin\frac{|\pi_1^a|}{F_0} \right]. \tag{13}$$

The effective potential (10), (13) will be used to search for CSB and for the derivation of meson masses.

B. Mass-gap equations and second variations

Let us now investigate the possible appearance of a non-zero v.e.v.'s of pseudoscalar fields. In order not to violate charge conservation, we may expect in the pseudoscalar channel the condensation of the neutral isospin components only,

$$\langle \pi_1^a \rangle = \langle \pi^0 \rangle \delta^{0a}, \qquad \langle \pi_2^a \rangle = \rho \delta^{0a}. \tag{14}$$

Now one obtains four mass-gap equations as, unlike in the chiral limit, the pion field becomes in principle observable

$$\begin{aligned}
&2(\Delta_{11}\sigma_1 + \Delta_{12}\sigma_2) + 2m_q d_1 \cos\frac{\langle \pi^0 \rangle}{F_0} = 4\lambda_1\sigma_1^3 + 3\lambda_5\sigma_1^2\sigma_2 \\
&+2(\lambda_3+\lambda_4)\sigma_1\sigma_2^2 + \lambda_6\sigma_2^3 + \rho^2\Big(2(\lambda_3-\lambda_4)\sigma_1 + \lambda_6\sigma_2\Big), \qquad (15)\\
&2(\Delta_{12}\sigma_1 + \Delta_{22}\sigma_2) + 2m_q d_2 \cos\frac{\langle \pi^0 \rangle}{F_0} = \lambda_5\sigma_1^3 + 2(\lambda_3+\lambda_4)\sigma_1^2\sigma_2 \\
&+3\lambda_6\sigma_1\sigma_2^2 + 4\lambda_2\sigma_2^3 + \rho^2\Big(\lambda_6\sigma_1 + 4\lambda_2\sigma_2\Big), \qquad (16)\\
&(d_1\sigma_1 + d_2\sigma_2)\sin\frac{\langle \pi^0 \rangle}{F_0} = -d_2\rho\cos\frac{\langle \pi^0 \rangle}{F_0}, \qquad (17)\\
&-m_q d_2 \sin\frac{\langle \pi^0 \rangle}{F_0} = \rho\,\frac{m_q d_2^2}{(d_1\sigma_1 + d_2\sigma_2)}\cos\frac{\langle \pi^0 \rangle}{F_0} \\
&= \rho\Big(- \Delta_{22} + (\lambda_3-\lambda_4)\sigma_1^2 + \lambda_6\sigma_1\sigma_2 + 2\lambda_2\sigma_2^2 + 2\lambda_2\rho^2\Big), \qquad (18)
\end{aligned}$$

As follows from eq. (17), the equality $\rho = 0$ entails $\langle \pi^0 \rangle = 0$ if $d_1\sigma_1 + d_2\sigma_2 \neq 0$ and $d_2 \neq 0$. However, as will be seen below, the combination $d_1\sigma_1 + d_2\sigma_2$ is related to the quark condensate

$$d_1\sigma_1 + d_2\sigma_2 = -\langle \bar{q}q \rangle > 0, \tag{19}$$

hence, this combination cannot be zero. For $d_2 = 0$ one has always $\langle \pi^0 \rangle = 0$ and the parity breaking pattern remains the same as for the massless case. For $d_2 \neq 0$ both pseudoscalar v.e.v. $\langle \pi^0 \rangle$ and ρ can arise simultaneously only. To avoid spontaneous parity breaking in then normal vacuum of QCD, it is thus necessary and sufficient to impose

$$(\lambda_3-\lambda_4)\sigma_1^2 + \lambda_6\sigma_1\sigma_2 + 2\lambda_2\sigma_2^2 > \Delta_{22} + \frac{m_q d_2^2}{(d_1\sigma_1 + d_2\sigma_2)}. \tag{20}$$

Since QCD in normal conditions does not lead to parity breaking, the low-energy model must necessarily fulfill (20).

In the usual vacuum state the necessary condition to have a minimum for non-zero σ_j (for vanishing ρ), equivalent to the condition of having CSB in QCD, can be derived from the condition to get a local maximum (or at least a saddle point) for zero σ_j. At this point the extremum is characterized by the matrix $-\Delta_{jk}$ in (5). It must have at least one negative eigenvalue. This happens either for $\mathrm{Det}\Delta > 0,\ \mathrm{tr}\{\Delta\} > 0$ (maximum at the origin) or for $\mathrm{Det}\Delta < 0$ (saddle point at the origin) . The sufficient conditions follow from the positivity of the second variation for a non-trivial solution of the two first equations (15), (16) at $\rho = 0$. The matrix containing the second variations $\hat{V}^{(2)}$ for the scalar sector is

$$\begin{aligned}
\frac{1}{2}V_{11}^{(2)\sigma} &= -\Delta_{11} + 6\lambda_1\sigma_1^2 + 3\lambda_5\sigma_1\sigma_2 + (\lambda_3+\lambda_4)\sigma_2^2, \\
V_{12}^{(2)\sigma} &= -2\Delta_{12} + 3\lambda_5\sigma_1^2 + 4(\lambda_3+\lambda_4)\sigma_1\sigma_2 + 3\lambda_6\sigma_2^2, \\
\frac{1}{2}V_{22}^{(2)\sigma} &= -\Delta_{22} + (\lambda_3+\lambda_4)\sigma_1^2 + 3\lambda_6\sigma_1\sigma_2 + 6\lambda_2\sigma_2^2. \qquad (21)
\end{aligned}$$

In turn, the nonzero elements of the second variations $\hat{V}^{(2)}$ in the pseudoscalar sector are

$$\frac{1}{2}(V_{11}^{(2)\pi})^{ab} = \delta^{ab} m_q \frac{d_1\sigma_1 + d_2\sigma_2}{F_0^2},$$
$$(V_{12}^{(2)\pi})^{ab} = \delta^{ab} \frac{2m_q d_2}{F_0},$$
$$\frac{1}{2}(V_{22}^{(2)\pi})^{ab} = \delta^{ab}\Big(- \Delta_{22} + (\lambda_3 - \lambda_4)\sigma_1^2 + \lambda_6\sigma_1\sigma_2 + 2\lambda_2\sigma_2^2\Big). \tag{22}$$

The required conditions are given by $\mathrm{tr}\left\{\hat{V}^{(2)}\right\} > 0$ and $\mathrm{Det}\hat{V}^{(2)} > 0$. For positive matrices it means that

$$V_{jj}^{(2)\sigma} > 0; \quad V_{kk}^{(2)\pi} > 0. \tag{23}$$

The eigenvalues of (22) eventually give the masses squared of π, π' mesons and thereby must be positive according to the inequality (20). The latter corresponds to the positivity of the determinant. From the positivity of matrix element $V_{11}^{(2)\pi}$ it follows that the combination $d_1\sigma_1 + d_2\sigma_2 > 0$. In fact, to the leading order in m_q the masses of π and π' mesons are proportional to $V_{11}^{(2)\pi}$ and $V_{22}^{(2)\pi}$, respectively, whereas the off-diagonal matrix element $V_{12}^{(2)\pi}$ contributes to the second order in m_q. The requirement to have a positive determinant of the matrix $V_{jk}^{(2)\pi}$ is exactly equivalent to (20) .

The two set of conditions, namely those presented in eq. (20) and in eq. (23) represent restrictions that the symmetry breaking pattern of QCD imposes on its low-energy effective realization at vanishing chemical potential.

One can easily find the correction linear in m_q to the vacuum solution in the chiral limit

$$\sigma_j(m_q) \simeq \sigma_j(0) + 2m_q\Delta_j^\sigma; \tag{24}$$
$$\vec{\Delta}^\sigma \equiv \begin{pmatrix} \Delta_1^\sigma \\ \Delta_2^\sigma \end{pmatrix} = \left(\hat{V}^{(2)\sigma}\right)^{-1} \cdot \vec{d} = \frac{1}{\mathrm{Det}\hat{V}^{(2)\sigma}} \begin{pmatrix} d_1 V_{22}^{(2)\sigma} - d_2 V_{12}^{(2)\sigma} \\ d_2 V_{11}^{(2)\sigma} - d_1 V_{12}^{(2)\sigma} \end{pmatrix}; \quad \vec{d} \equiv \begin{pmatrix} d_1 \\ d_2 \end{pmatrix}.$$

Using these equations the corrections to the masses of scalar and heavy pseudoscalar mesons can be derived straightforwardly. In particular, for scalar mesons the corrections to the mass matrix are

$$\begin{aligned} \Delta V_{kl}^{(2)\sigma} &= 2m_q \sum_{j,m=1,2} \partial_j V_{kl}^{(2)\sigma} \big(\hat{V}^{(2)\sigma}\big)^{-1}_{jm} d_m \\ &= 2m_q \sum_{j,m=1,2} \partial_k V_{lj}^{(2)\sigma} \big(\hat{V}^{(2)\sigma}\big)^{-1}_{jm} d_m = 2m_q \partial_k \Big(\hat{V}^{(2)\sigma}\Big) \cdot \Big(\hat{V}^{(2)\sigma}\Big)^{-1} \cdot \vec{d}; \quad \partial_j \equiv \partial_{\sigma_j}, \end{aligned} \tag{25}$$

whereas in the pseudoscalar sector

$$\Delta V_{22}^{(2)\pi} = 2m_q \sum_{j,m=1,2} \partial_j V_{22}^{(2)\pi} \big(\hat{V}^{(2)\sigma}\big)^{-1}_{jm} d_m . \tag{26}$$

The latter term saturates the current quark mass correction for heavy pseudoscalar meson masses.

IV. REDUCTION OF COUPLING CONSTANTS AND EXTREMA OF V_{eff}

Let us investigate how many extrema the effective potential possesses for different values of the coupling constants. In this section we take the chiral limit $m_q = 0$ for simplicity. It turns out that when the chemical potential and temperature are zero one can eliminate one of the constant in the effective potential by a redefinition of the fields. Indeed, one can change the variable

$$H_2 = \alpha H_1 + \beta \tilde{H}_2, \tag{27}$$

using a linear transformation with real coefficients α, β (to preserve reality of $\tilde{\sigma}_j, \pi_j^a$). With the help of this redefinition one can diagonalize the quadratic part in (5) and make its coefficients equal $\tilde{\Delta}_{11} = \tilde{\Delta}_{22} =$

$\det \hat{\Delta}/\Delta_{22} \equiv \Delta$. Then

$$\sum_{j,k=1}^{2} \mathrm{tr}\left\{H_j^\dagger \Delta_{jk} H_k\right\} = \Delta \mathrm{tr}\left\{H_1^\dagger H_1 + \tilde{H}_2^\dagger \tilde{H}_2\right\}. \tag{28}$$

A further reduction of the coupling constants affects the dependence of free energy on finite chemical potential and temperature (see below), but it can be implemented when both external control parameters vanish; namely we perform the following orthogonal rotation of two fields

$$H_1 = \cos\phi \check{H}_1 + \sin\phi \check{H}_2,\ \tilde{H}_2 = -\sin\phi \check{H}_1 + \cos\phi \check{H}_2. \tag{29}$$

Then the coefficient in the operator $(\check{H}_1^\dagger \check{H}_2 + \check{H}_2^\dagger \check{H}_1)\check{H}_1^\dagger \check{H}_1$ becomes equal to

$$\begin{aligned} \frac{\check{\lambda}_5}{\cos^4\phi} &= \lambda_5 - 2(\lambda_3+\lambda_4-2\lambda_1)\tan\phi - 3(\lambda_5-\lambda_6)\tan^2\phi \\ &\quad +2(\lambda_3+\lambda_4-2\lambda_2)\tan^3\phi - \lambda_6\tan^4\phi \equiv \mathcal{P}_{\lambda_5}(\tan\phi). \end{aligned} \tag{30}$$

One can always fix $\lambda_6 < 0$ by reflection of $\check{H}_2$. Then if $\lambda_5 < 0$ then $\mathcal{P}_{\lambda_5}(0) < 0$ but evidently for $\tan\phi \gg 1$, $\mathcal{P}_{\lambda_5}(\tan\phi) \sim -\lambda_6 \tan^4\phi > 0$ and therefore the equation $\mathcal{P}_{\lambda_5}(\tan\phi) = 0$ has at least one (positive) real root . In the complementary region $\lambda_5 \geq 0$ and therefore $\mathcal{P}_{\lambda_5}(0) > 0$. In this case one can look at $\tan\phi = \pm 1$ where

$$\mathcal{P}_{\lambda_5}(\pm 1) = -2(\lambda_5 - \lambda_6) \pm 4(\lambda_1 - \lambda_2), \tag{31}$$

so that one of these combinations is negative. Again the comparison with the asymptotics allows to conclude that there is a real root for $\mathcal{P}_{\lambda_5}(\tan\phi) = 0$. Thus for any sign of λ_5 it can be eliminated by a proper rotation of scalar fields.

Let us take the basis of operators with $\check{\lambda}_5 = 0$. Then, after renaming the fields

$$\begin{aligned} V_{\text{eff}} &= -\Delta\Big((\sigma_1)^2 + (\sigma_2)^2\Big) + \lambda_2\Big((\pi_2^a)^2\Big)^2 + (\pi_2^a)^2\Big(-\Delta + (\lambda_3 - \lambda_4)\sigma_1^2 + \lambda_6\sigma_1\sigma_2 + 2\lambda_2\sigma_2^2\Big) \\ &\quad +\lambda_1\sigma_1^4 + \lambda_2\sigma_2^4 + (\lambda_3+\lambda_4)\sigma_1^2\sigma_2^2 + \lambda_6\sigma_1\sigma_2^3. \end{aligned} \tag{32}$$

This potential simplifies the mass gap equations and second variations in order to investigate their solutions analytically. The effective potential must provide the familiar CSB at normal conditions ($\mu = T = 0$). Thus in the chiral limit there are at least two minima related by the symmetry rotation $\sigma_{1,2} \to -\sigma_{1,2}$ and one maximum at the origin. This is implemented by assigning a real singlet v.e.v. $\langle\sigma_1\rangle > 0$ to $\check{H}_1$ thereby selecting one of the minima.

In this section we shall assume $\lambda_5 = 0$ in order to determine the different vacua of the theory at zero temperature and chemical potential.

A. Search for the extrema of the effective potential

In the parity symmetric case the second (16) reads

$$\sigma_2(-\Delta + (\lambda_3+\lambda_4)\sigma_1^2 + \frac{3}{2}\lambda_6\sigma_1\sigma_2 + 2\lambda_2\sigma_2^2) = 0. \tag{33}$$

One solution is

$$\sigma_2^{(0)} = 0, \quad (\sigma_1^{(0)})^2 = \frac{\Delta}{2\lambda_1}, \tag{34}$$

from eq. (15). For stable solutions $\lambda_1 > 0$ and therefore $\Delta > 0$.

Another set of solutions $\sigma_{1,2}^{(a)}$; $a = 1,2,3$ comes from eq. (33) for $\sigma_2 \neq 0$. With a combination of the mass-gap eqs. (15) and (33) one can decouple the equation in terms of the ratio $t = \sigma_2/\sigma_1$,

$$\begin{aligned} &\mathcal{P}_3(t) = t^3 - at^2 - bt + c = 0, \\ &a = \frac{2\big((\lambda_3+\lambda_4) - 2\lambda_2\big)}{-\lambda_6}, \quad b = 3, \quad c = \frac{2\big((\lambda_3+\lambda_4) - 2\lambda_1\big)}{-\lambda_6}, \end{aligned} \tag{35}$$

where the sign is fixed for $\lambda_6 < 0$ and $c > 0$ as it is shown in the next subsection. As the order of the equation is odd there may one or three real solutions. Because $\mathcal{P}_3(0) > 0,\ \mathcal{P}_3'(0) < 0$ one concludes that one of the solutions is negative. Let us analyze the extrema of $\mathcal{P}_3(t)$

$$\mathcal{P}_3'(t) = 0 \longrightarrow t^2 - \frac{2}{3}at - b = 0, \quad t_\pm = \frac{1}{3}a \pm \sqrt{\frac{1}{9}a^2 + b},\ t_+ > 0,\ t_- < 0. \tag{36}$$

All together it means that a negative solution $t^{(1)} < 0$ always exists and (if any) two more solutions are positive, $t^{(2)} < t^{(3)}$, and separated by a minimum. Therefore the existence of two positive solutions is regulated by the sign of $\mathcal{P}_3(t_+)$. They exist if

$$\mathcal{P}_3(t_+) = -\frac{2}{27}a^3 - \frac{1}{3}ab + c - \frac{2}{9}a^2\sqrt{\frac{1}{9}a^2 + b} = c - \frac{a}{3}\left(\frac{a}{3} + \sqrt{\frac{a^2}{9} + b}\right)^2 < 0. \tag{37}$$

Evidently it takes place for positive a, i.e. for

$$(\lambda_3 + \lambda_4) > 2\lambda_2. \tag{38}$$

Later on we will see that in order to implement a first-order phase transition to stable nuclear matter we

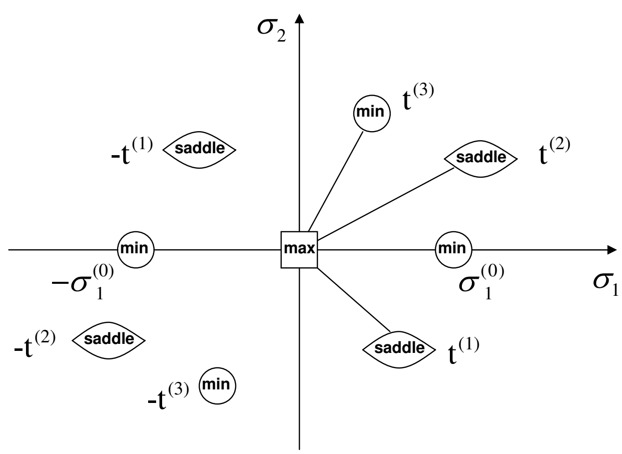

FIG. 1: *Extrema of effective potential in the reduction basis: the maximum is placed in the square, up to four minima are located in the circles and the corresponding four saddle points are depicted by the lentils. The existence of two solutions $t^{(2)}, t^{(3)}$ with positive values of σ_j is governed by condition (37). Which one corresponds to the true minimum depends on the actual value of the phenomenological constants.*

need two minima for positive σ_1 which entails two more minima for negative σ_j due to symmetry under $\sigma_j \to -\sigma_j$ and therefore four saddle points must connect them. Thereby, in the half plane of positive σ_1 one has to reveal four solutions, namely, one is $\sigma_2^{(0)} = 0$ and three for $\sigma_2 \neq 0$ so the condition (37) should be satisfied in order to be able to describe the saturation point, see Fig.1 .

Let us recall that all the inequalities obtained in this section are referred to the field basis with fully diagonal $\Delta_{ij} = \Delta\delta_{ij}$ and with $\lambda_5 = 0$.

B. Selection of the minima

In all cases the condition of minimum come from the positive definiteness of the matrix of second variations

$$
\begin{aligned}
\frac{1}{2}V^{(2)}_{\sigma_1\sigma_1} &= -\Delta + 6\lambda_1\sigma_1^2 + (\lambda_3+\lambda_4)\sigma_2^2 > 0,\\
V^{(2)}_{\sigma_1\sigma_2} &= 4(\lambda_3+\lambda_4)\sigma_1\sigma_2 + 3\lambda_6\sigma_2^2,\\
\frac{1}{2}V^{(2)}_{\sigma_2\sigma_2} &= -\Delta + (\lambda_3+\lambda_4)\sigma_1^2 + 3\lambda_6\sigma_1\sigma_2 + 6\lambda_2\sigma_2^2 > 0,\\
\frac{1}{2}V^{(2)}_{\pi_2\pi_2} &= -\Delta + (\lambda_3-\lambda_4)\sigma_1^2 + \lambda_6\sigma_1\sigma_2 + 2\lambda_2\sigma_2^2 > 0.
\end{aligned} \tag{39}
$$

For $\sigma_2^{(0)} = 0$ they read

$$(\lambda_3 \pm \lambda_4) > 2\lambda_1, \longrightarrow \lambda_3 > |\lambda_4| \tag{40}$$

for $\Delta > 0$ as it is required by the absence of chiral collapse (and the spectrum at the SPB point, see below). It gives support to the condition $c > 0$ in the previous subsection.

For $\sigma_2 \neq 0$ one obtains a number of bounds on the solution from the second variation

$$
\begin{aligned}
\frac{1}{2}V^{(2)}_{\sigma_1\sigma_1} &= (\sigma_1)^2\Big[4\lambda_1 - \frac{1}{2}\lambda_6 t^3\Big] > 0,\\
V^{(2)}_{\sigma_1\sigma_2} &= (\sigma_1)^2\Big[4(\lambda_3+\lambda_4)t + 3\lambda_6 t^2\Big],\\
\frac{1}{2}V^{(2)}_{\sigma_2\sigma_2} &= (\sigma_1)^2\Big[\frac{3}{2}\lambda_6 t + 4\lambda_2 t^2\Big] > 0,\\
\frac{1}{2}V^{(2)}_{\pi_2\pi_2} &= (\sigma_1)^2\Big[-2\lambda_4 - \frac{1}{2}\lambda_6 t\Big] > 0.
\end{aligned} \tag{41}
$$

Evidently if $\lambda_4 > 0$ then $\sigma_2 > 0 \to t > 0$. The remaining bound must come from the positivity, $\det \hat{V}^{(2)} > 0$.

V. FINITE TEMPERATURE AND BARYON CHEMICAL POTENTIAL

A. Coupling the effective lagrangian to the environment

We are building a model of meson medium starting from the quark sector of QCD. Its thermodynamical properties and relationship to a dense baryon matter will be examined with the help of thermodynamical potentials derived from the constituent quark model in the mean field (large N_c) approach. This gives a prescription to connect the properties of quark and nuclear matter and estimate the parameters of our model to reproduce meson phenomenology and the bulk characteristics of nuclear matter such as binding energy, normal nuclear density and (in)compressibility.

The meson degrees of freedom present in our model appear after bosonization of QCD in the vacuum. The effects of infinite homogeneous baryon matter on the effective meson lagrangian is described by the baryon chemical potential μ, which is transmitted to the meson lagrangian via a local quark-meson coupling (in the leading order of chiral expansion μ^2/Λ^2). In turn, in the large N_c limit one can neglect the temperature dependence due to meson collisions. The temperature T is induced with the help of the imaginary time Matsubara formalism for quark Green functions[39]

$$\omega_n = \frac{(2n+1)\pi}{\beta}, \quad \beta = \frac{1}{kT}. \tag{42}$$

For real physics with 3 colors this approximation to thermal properties of mesons is expected to be less precise as meson loops contribute substantially to the thermodynamic characteristics. Nevertheless it should be sufficient to describe qualitatively the interplay between baryon density and temperature at the phase transition.

Without loss of generality we can regard one of the collective fields H_j, namely, H_1 as the one having local coupling to quarks: this actually defines the chiral multiplet H_1. The set of coupling constants in (5) is sufficient to support this choice as well as to fix the Yukawa coupling constant to unity.

We notice that this specification of collective fields is compatible with the transformation (27) and therefore one can deal with the diagonal quadratic part of the potential. However the additional linear transformation (29) would split the constituent mass in the quark Yukawa vertex into two fields $\bar{q}_R H_1 q_L + \text{h. c.} \longrightarrow \bar{q}_R(\cos\phi \tilde{H}_1 + \sin\phi \tilde{H}_2) q_L + \text{h. c.}$ It means that a possible change of the basis used in sec. 4 to eliminate the constant λ_5 affects the chemical potential driver $\Delta V_{eff}(\sigma_1, \beta, \mu) \to \Delta V_{eff}(\sqrt{(\cos\phi\ \tilde{\sigma}_1 + \sin\phi\ \tilde{\sigma}_2)^2 + \tilde{\rho}^2},\ \beta, \mu)$. Thereby all the mass gap equations (15)–(18) obtain new contributions depending on T and μ.

To keep the consideration more tractable we do not reduce the coupling $\lambda_5 \to 0$ but rather select the basis in which finite density and temperature is transmitted to the boson sector by means of

$$\Delta\mathcal{L} = -(\bar{q}_R H_1 q_L + \bar{q}_L H_1^\dagger q_R) \longrightarrow -\bar{Q}\sigma_1 Q, \tag{43}$$

where $Q_L = \xi q_L, Q_R = \xi^\dagger q_R$ stand for constituent quarks [23]. However, the results derived in the previous section on the different vacua for vanishing temperature and chemical potential remain obviously valid.

Following this recipe the quark temperature and chemical potential dependence can be derived for mass gap equations – the conditions for a minimum of the effective potential. Namely, taking into account the transformation (29) the first equation (15) is modified to

$$\begin{aligned}
&2\Delta\sigma_1 + 2m_q d_1 \cos\frac{\langle\pi^0\rangle}{F_0} = 4\lambda_1\sigma_1^3 + 3\lambda_5\sigma_1^2\sigma_2 + 2(\lambda_3+\lambda_4)\sigma_1\sigma_2^2 + \lambda_6\sigma_2^3 \\
&+\rho^2\Big(2(\lambda_3-\lambda_4)\sigma_1 + \lambda_6\sigma_2\Big) + 2\mathcal{N}\sigma_1\mathcal{A}(\sigma_1,\mu,\beta),
\end{aligned} \tag{44}$$

with notation $\mathcal{N} \equiv \frac{N_c N_f}{4\pi^2}$. In turn

$$\begin{aligned}
\mathcal{A}(\sigma_1,\mu,\beta) &= \frac{1}{\beta}\sum_n \exp(i\omega_n\eta)\int\frac{d^3p}{\pi}\frac{1}{(i\omega_n+\mu)^2 - E^2} - [T=0,\ \mu=0] \\
&= \int\frac{d^3p}{2\pi E}\ (f(E-\mu) - f(-E-\mu) + 1) \\
&= 2\int_{\sigma_1}^{\infty} dE\ \sqrt{E^2-\sigma_1^2}\ (f(E-\mu) - f(-E-\mu) + 1) \\
&= 2\int_{\sigma_1}^{\infty} dE\ \sqrt{E^2-\sigma_1^2}\ \frac{\cosh(\beta\mu) + \exp(-\beta E)}{\cosh(\beta\mu)+\cosh(\beta E)}\ ,
\end{aligned} \tag{45}$$

where $E = \sqrt{p^2+\sigma_1^2}$ and the limit $\eta \to 0$ is to be taken after the Matsubara summation and the Fermi distribution function is introduced

$$f(x) = \frac{1}{\exp(\beta x)+1}\ . \tag{46}$$

The corresponding one-quark loop contribution to V_{eff} can be obtained by integration

$$\begin{aligned}
\Delta V_{\text{eff}}\ (\sigma_1,\mu,\beta) &= -2\mathcal{N}\int_{\sigma_1}^{\infty} dx\, x\mathcal{A}(x,\mu,\beta) \\
&= -\frac{4}{3}\mathcal{N}\int_{\sigma_1}^{\infty} dE\Big(E^2-\sigma_1^2\Big)^{3/2}\frac{\cosh(\beta\mu)+\exp(-\beta E)}{\cosh(\beta\mu)+\cosh(\beta E)},
\end{aligned} \tag{47}$$

so that it vanishes for very large σ_1.

Using the gap equations and (44), the value of the effective potential at its minima is given by the compact

expression

$$V_{\text{eff}}(\mu,\beta) = -\frac{1}{2}\Delta\Big(\langle\sigma_1\rangle^2 + \langle\sigma_2\rangle^2 + \rho^2\Big) + \frac{3}{2}m_q\left[-(d_1\langle\sigma_1\rangle + d_2\langle\sigma_2\rangle)\cos\frac{\langle\pi^0\rangle}{F_0} + d_2\rho\sin\frac{\langle\pi^0\rangle}{F_0}\right]$$
$$+\,\Delta V_{\text{eff}}\,(\langle\sigma_1\rangle,\mu,\beta) - \frac{1}{2}\mathcal{N}\langle\sigma_1\rangle^2(\mu,\beta)\mathcal{A}(\langle\sigma_1\rangle,\mu,\beta); \tag{48}$$
$$\Delta V_{\text{eff}}\, - \frac{1}{2}\mathcal{N}\langle\sigma_1\rangle^2\mathcal{A} = -\frac{1}{3}\mathcal{N}\int\limits_{\langle\sigma_1\rangle}^{\infty} dE(4E^2 - \langle\sigma_1\rangle^2)\Big(E^2 - \langle\sigma_1\rangle^2\Big)^{1/2}\frac{\cosh(\beta\mu) + \exp(-\beta E)}{\cosh(\beta\mu) + \cosh(\beta E)}.$$

It should be emphasized that all the above results have corrections of $\mathcal{O}\left(\mu^2/\Lambda^2, \sigma_1^2/\Lambda^2\right)$. In the remaining part of this section we simplify our analysis and take the chiral limit $m_q = 0$.

B. Thermodynamic properties of the model at $T \neq 0$

Thermodynamically the system is described by the pressure P, the energy density, ε and the entropy density s. The pressure is determined by the potential density difference with and without the presence of chemical potential, $dP = -dV$

$$P(\sigma_j(\mu,\beta),\mu,\beta) \equiv V_{\text{eff}}\,\Big(\sigma_j(0,0)\Big) - V_{\text{eff}}\,\Big(\sigma_j(\mu,\beta),\rho(\mu,\beta),\mu,\beta\Big), \tag{49}$$

where the dependence of solutions of mass gap eqs. σ_j and ρ on μ and β has been shown explicitly. The energy density is related to the pressure, density and entropy density by

$$\varepsilon = -P + N_c\mu\varrho_B + Ts. \tag{50}$$

The chemical potential is defined as

$$\partial_{\varrho_B}\varepsilon = N_c\mu, \tag{51}$$

with the entropy and volume held fixed. The factor N_c is introduced to relate the quark and baryon chemical potentials. Since ε is independent of μ

$$\partial_\mu P = N_c\varrho_B, \quad \varrho_B = -\frac{1}{N_c}\partial_\mu V_{\text{eff}}\, = \frac{N_f}{\pi^2}\int\limits_{\sigma_1}^{\infty} dE\; E\Big(E^2 - \sigma_1^2\Big)^{1/2}\frac{\sinh(\beta\mu)}{\cosh(\beta\mu) + \cosh(\beta E)}. \tag{52}$$

In turn the entropy is defined as

$$s = \partial_T P = -\partial_T V_{\text{eff}}\,, \quad Ts = \beta\partial_\beta V_{\text{eff}}. \tag{53}$$

Let us relate the last two terms in (48) and (50)

$$\Delta V_{\text{eff}}\,(\sigma_1,\mu,\beta) - \frac{1}{2}\mathcal{N}\sigma_1^2(\mu,\beta)\mathcal{A}(\sigma_1,\mu,\beta) = (1 - \frac{1}{4}\sigma_1\partial_{\sigma_1})\Delta V_{\text{eff}}\,(\sigma_1,\mu,\beta)$$
$$= \frac{1}{4}(\mu\partial_\mu - \beta\partial_\beta)\Delta V_{\text{eff}}\,(\sigma_1,\mu,\beta) = \frac{1}{4}(\mu\partial_\mu - \beta\partial_\beta)V_{\text{eff}}\,(\sigma_1,\mu,\beta) = -\frac{1}{4}(N_c\mu\varrho_B + Ts), \tag{54}$$

The above equation allows to calculate the entropy in our model.

C. Zero temperature and finite density

In this subsection we consider the zero-temperature case and study the regime of chemical potentials comparable with the v.e.v. σ_1. At zero temperature $T = 0$ the contribution from μ to the effective potential is

$$\Delta V_{\text{eff}}\,(\sigma_1,\mu) \;=\; \frac{\mathcal{N}}{2}\theta(\mu - \sigma_1)\Bigg[\mu\sigma_1^2\sqrt{\mu^2 - \sigma_1^2} - \frac{2\mu}{3}(\mu^2 - \sigma_1^2)^{3/2}$$
$$-\sigma_1^4\ln\frac{\mu + \sqrt{\mu^2 - \sigma_1^2}}{\sigma_1}\Bigg]\left(1 + O\left(\frac{\mu^2}{\Lambda^2};\frac{\sigma_1^2}{\Lambda^2}\right)\right). \tag{55}$$

The total value of the effective potential at its minimum is

$$V_{\text{eff}}(\mu) = -\frac{1}{2}\Delta\Big(\sigma_1^2(\mu) + \sigma_2^2(\mu) + \rho^2(\mu)\Big) - \frac{\mathcal{N}}{3}\mu\Big(\mu^2 - \sigma_1(\mu)\Big)^{3/2}\theta\Big(\mu - \sigma_1(\mu)\Big). \tag{56}$$

Higher-order terms of the chiral expansion in $1/\Lambda^2$ are not considered and the gluon condensate contribution is assumed to be hidden in the dynamical mass value.

Accordingly in the first mass gap equation (44)

$$\begin{aligned}\mathcal{A}(\sigma_1,\mu,\beta) &\overset{\beta\to\infty}{=} 2\theta(\mu-\sigma_1)\int_{\sigma_1}^{\mu} dE\,\sqrt{E^2-\sigma_1^2}\\ &= \theta(\mu-\sigma_1)\Big[\mu\sqrt{\mu^2-\sigma_1^2} - \sigma_1^2\ln\frac{\mu+\sqrt{\mu^2-\sigma_1^2}}{\sigma_1}\Big].\end{aligned} \tag{57}$$

Then the second variation of effective potential is modified in the only element

$$\begin{aligned}\frac{1}{2}V_{11}^{(2)\sigma} &= -\Delta_{11} + 6\lambda_1\sigma_1^2 + 3\lambda_5\sigma_1\sigma_2 + (\lambda_3+\lambda_4)\sigma_2^2\\ &\quad +\mathcal{N}\theta(\mu-\sigma_1)\left[\mu\sqrt{\mu^2-\sigma_1^2} - 3\sigma_1^2\ln\frac{\mu+\sqrt{\mu^2-\sigma_1^2}}{\sigma_1}\right].\end{aligned} \tag{58}$$

The effective potential (55),(56) is normalized to reproduce the baryon density for quark matter

$$\varrho_B = -\frac{1}{N_c}\partial_\mu\Delta V_{\text{eff}}\,(\sigma_1,\mu)\Big|_{\sigma_1(\mu)} = -\frac{1}{N_c}\frac{\mathrm{d}V_{\text{eff}}(\mu)}{\mathrm{d}\mu} = \frac{N_f}{3\pi^2}p_F^3 = \frac{N_f}{3\pi^2}\Big(\mu^2-\sigma_1^2(\mu)\Big)^{3/2}, \tag{59}$$

where the quark Fermi momentum is $p_F = \sqrt{\mu^2-\sigma_1^2(\mu)}$.

VI. SPONTANEOUS PARITY BREAKING PHASE

A. Mass gap and critical lines for the SPB transition

Let us examine the possible existence of a critical point, in the chiral limit $m_q = 0$ for simplicity, where the strict inequality (20) does not hold and instead for $\mu \geq \mu_{crit}$

$$(\lambda_3-\lambda_4)\sigma_1^2 + \lambda_6\sigma_1\sigma_2 + 2\lambda_2\Big(\sigma_2^2+\rho^2\Big) = \Delta. \tag{60}$$

After substituting Δ from (60) into the second eq. (15) one finds that

$$\lambda_5\sigma_1^2 + 4\lambda_4\sigma_1\sigma_2 + \lambda_6\Big(\sigma_2^2+\rho^2\Big) = 0, \tag{61}$$

where we have taken into account that $\sigma_1 \neq 0$. This, together with (60) completely fixes the v.e.v.'s of the scalar fields $\sigma_{1,2}$. If $\lambda_2 = 0$ and/or $\lambda_6 = 0$ equations (60) or (61) unambiguously determine the relation between σ_1 and σ_2. Otherwise if $\lambda_2\lambda_6 \neq 0$ these two equations still allow to get rid of the v.e.v. of pseudoscalar field leading to the relation

$$\Big(2\lambda_5\lambda_2 + \lambda_6(\lambda_4-\lambda_3)\Big)\sigma_1^2 + \Big(8\lambda_2\lambda_4 - \lambda_6^2\Big)\sigma_1\sigma_2 = -\lambda_6\Delta, \tag{62}$$

whose solution is

$$\sigma_2 = A\sigma_1 + \frac{B}{\sigma_1} > 0; \qquad A \equiv \frac{2\lambda_5\lambda_2 + \lambda_6(\lambda_4-\lambda_3)}{\lambda_6^2 - 8\lambda_2\lambda_4}; \qquad B \equiv \frac{\lambda_6\Delta}{\lambda_6^2-8\lambda_2\lambda_4}. \tag{63}$$

Thus in the parity breaking phase the relation between the two scalar v.e.v's is completely determined and, in particular, does not depend neither on ρ nor on μ.

The first mass gap equation (44) can be brought to the form

$$\Delta = 2\lambda_1\sigma_1^2 + \lambda_5\sigma_1\sigma_2 + (\lambda_3-\lambda_4)(\sigma_2^2+\rho^2) + \mathcal{N}\mathcal{A}(\sigma_1,\mu,\beta), \tag{64}$$

if one employs eq. (61). Together with eq. (63) it allows to find all v.e.v.'s of the scalar fields σ_j, ρ as functions of temperature and chemical potential.

Let us now find the critical value of the chemical potential, namely the value where $\rho(\mu_c) = 0$, but equations (60), (61), (63) hold. Combining the two equations (60), (61)

$$\lambda_6 r^2 + 4\lambda_4 r + \lambda_5 = 0; \quad r \equiv \frac{\sigma_2}{\sigma_1}. \tag{65}$$

In order for a SPB phase to exist this equation has to possess real solutions. If $\lambda_6 = 0$ there is only one solution corresponding to a second order transition, but there may exist other solutions that fall beyond the accuracy of our low energy model (which becomes inappropriate for small values of σ_1).

We stress that equations (63) and (65) contain only the structural constants of the potential and do not depend on temperature or chemical potential manifestly. Thus with the help of

$$r_{crit} = r_{\pm} = \frac{-2\lambda_4 \pm \sqrt{4\lambda_4^2 - \lambda_5\lambda_6}}{\lambda_6} \tag{66}$$

one can immediately calculate

$$\sigma_1^{\pm}(\Delta, \lambda_j) = \sqrt{\frac{B}{r_{\pm} - A}}; \qquad \sigma_2^{\pm}(\Delta, \lambda_j) = r_{\pm}\sigma_1^{\pm}, \tag{67}$$

where $\sigma_i^{\pm}$ are the corresponding critical values.

After substituting these values into equation (64) for each critical set of $\sigma_{1,2}$ one derives the boundary of the parity breaking phase

$$\mathcal{N}\mathcal{A}(\sigma_1^{\pm}, \mu_{crit}, \beta_{crit}) = \Delta - 2\lambda_1(\sigma_1^{\pm})^2 - \lambda_5\sigma_1^{\pm}\sigma_2^{\pm} - (\lambda_3 - \lambda_4)(\sigma_2^{\pm})^2. \tag{68}$$

It must be positive at critical values of $\sigma_j^{\pm}$. The relation (68) defines a strip in the $T - \mu$ plane where parity is spontaneously broken. From (45) one can obtain that $\mathcal{A} > 0$ and $\mathcal{A} \to \infty$ when $T, \mu \to \infty$. It means that for *any* nontrivial solution $r_{\pm}, \sigma_1^{\pm}, \sigma_2^{\pm}$ the parity breaking phase boundary exists.

Thus we have proved that if the phenomenon of parity breaking is realized for zero temperature it will take place in a strip including lower chemical potentials but higher temperatures.

B. Mass-gap equations in SPB beyond the chiral limit

Let us now examine again the possible existence of a critical point where the strict inequality (20) does not hold and for $\mu > \mu_{crit}$

$$\begin{aligned}(\lambda_3 - \lambda_4)\sigma_1^2 + \lambda_6\sigma_1\sigma_2 + 2\lambda_2\left(\sigma_2^2 + \rho^2\right) - \Delta &= \frac{m_q d_2^2}{(d_1\sigma_1 + d_2\sigma_2)} \cos\frac{\langle\pi^0\rangle}{F_0} \\ &= \frac{m_q d_2^2}{\sqrt{d_2^2\rho^2 + (d_1\sigma_1 + d_2\sigma_2)^2}},\end{aligned} \tag{69}$$

where the following consequence of equation(17) has been used:

$$\cos\frac{\langle\pi^0\rangle}{F_0} = \frac{d_1\sigma_1 + d_2\sigma_2}{\sqrt{d_2^2\rho^2 + (d_1\sigma_1 + d_2\sigma_2)^2}}. \tag{70}$$

When combining equation (69) with (15), (16) one finds that

$$\begin{aligned}&d_1\left(\lambda_5\sigma_1^2 + 4\lambda_4\sigma_1\sigma_2 + \lambda_6\left(\sigma_2^2 + \rho^2\right)\right) \\ &= 2d_2\left(-\Delta + 2\lambda_1\sigma_1^2 + \lambda_5\sigma_1\sigma_2 + (\lambda_3 - \lambda_4)\left(\sigma_2^2 + \rho^2\right) + \mathcal{N}\mathcal{A}(\sigma_1, \mu, \beta)\right),\end{aligned} \tag{71}$$

$$\begin{aligned}&d_2\left(\lambda_5\sigma_1^2 + 4\lambda_4\sigma_1\sigma_2 + \lambda_6\left(\sigma_2^2 + \rho^2\right)\right) \\ &= 2d_1\Big(-\Delta + (\lambda_3 - \lambda_4)\sigma_1^2 + \lambda_6\sigma_1\sigma_2 + 2\lambda_2\left(\sigma_2^2 + \rho^2\right)\Big),\end{aligned} \tag{72}$$

where we have taken into account that $\sigma_1 \neq 0$. These two relations determine the v.e.v.'s of the scalar fields $\sigma_{1,2}$. If $\lambda_2 = \lambda_6 = 0$ and/or $\lambda_3 = \lambda_4, \lambda_6 = 0$ equations (71) and (72) firmly fix the relation between σ_1 and σ_2. Otherwise an appropriate combination of these two equations still allows us to get rid of the v.e.v. of the pseudoscalar field[44]. Thus in the parity breaking phase the relation between the two scalar v.e.v's is completely determined and in particular does not depend neither on ρ nor on μ. Using equations (69), (71) and (72) one can easily eliminate the variables ρ and σ_2, obtaining an equation for the variable σ_1^2/μ^2 . The latter completes the determination of the v.e.v.s.

We notice that in the chiral limit $m_q \to 0$ the constants d_1, d_2 become arbitrary and therefore (71), (72) entail three independent relations coinciding with (60), (61), (64).

VII. APPROACHING THE SPB PHASE TRANSITION

Let us find the character of the phase transition to the SPB phase. For small values of $\mu - \sigma_1 > 0$, we know that the value of the odd parity condensate ρ is zero. Setting $\rho = 0$ in equations (15), (16), (44) and using (64) and differentiating w.r.t. μ we get

$$\sum_{k=1,2} \hat{V}^{(2)\sigma}_{jk} \partial_\mu \sigma_k = -4\mathcal{N}\sigma_1 \sqrt{\mu^2 - \sigma_1^2}\,\delta_{j1}, \tag{73}$$

or, after inversion of the matrix of second variations,

$$\partial_\mu \sigma_1 = -4\mathcal{N}\sigma_1 \sqrt{\mu^2 - \sigma_1^2}\frac{V^{(2)\sigma}_{22}}{\mathrm{Det}\hat{V}^{(2)\sigma}} < 0, \qquad \partial_\mu \sigma_2 = 4\mathcal{N}\sigma_1 \sqrt{\mu^2 - \sigma_1^2}\frac{V^{(2)\sigma}_{12}}{\mathrm{Det}\hat{V}^{(2)\sigma}}. \tag{74}$$

The possibility of SPB is controlled by the inequality (20); in order to approach a SPB phase transition when the chemical potential is increasing we have to diminish the l.h.s. of inequality (20) and therefore we need to have

$$\partial_\mu \Big[(\lambda_3 - \lambda_4)\sigma_1^2 + \lambda_6 \sigma_1 \sigma_2 + 2\lambda_2 \sigma_2^2 - \frac{m_q d_2^2}{(d_1\sigma_1 + d_2\sigma_2)} \Big] < 0. \tag{75}$$

This is equivalent (using (74)) to

$$\Big(\lambda_6 \sigma_1 + 4\lambda_2 \sigma_2 + \frac{m_q d_2^3}{(d_1\sigma_1 + d_2\sigma_2)^2} \Big) V^{(2)\sigma}_{12} < \Big(2(\lambda_3 - \lambda_4)\sigma_1 + \lambda_6\sigma_2 + \frac{m_q d_1 d_2^2}{(d_1\sigma_1 + d_2\sigma_2)^2} \Big) V^{(2)\sigma}_{22}. \tag{76}$$

This last inequality is a necessary condition that has to be satisfied by the model at zero chemical potential for it to be potentially capable of yielding SPB. Evidently, this inequality must hold across the critical point in order that $\partial_\mu \rho^2 > 0, \partial_\mu (\pi^0)^2 > 0$.

A. Second variations in the SPB phase and character of the phase transition in the chiral limit

Once a condensate for π_2^0 appears spontaneously the vector $SU(2)$ symmetry is broken to $U(1)$ and two charged π' mesons are expected to possess zero masses as dictated by the Goldstone theorem. For simplicity let us consider zero temperature. Then in the chiral limit the matrix of second variations $\hat{V}^{(2)} =$

$\left(V^{(2)}_{ab}\right)$; $a,b=1,2,0$; reads

$$
\begin{aligned}
\frac{1}{2}V^{(2)\sigma}_{11} &= -\Delta + 6\lambda_1\sigma_1^2 + 3\lambda_5\sigma_1\sigma_2 + (\lambda_3+\lambda_4)\sigma_2^2 + (\lambda_3-\lambda_4)\rho^2 \\
&\quad + \mathcal{N}\left[\mu\sqrt{\mu^2-\sigma_1^2} - 3\sigma_1^2\ln\frac{\mu+\sqrt{\mu^2-\sigma_1^2}}{\sigma_1}\right] \equiv \frac{1}{2}\mathcal{V}_{11}, \\
V^{(2)\sigma}_{12} &= 3\lambda_5\sigma_1^2 + 4(\lambda_3+\lambda_4)\sigma_1\sigma_2 + 3\lambda_6\sigma_2^2 + \lambda_6\rho^2 \equiv \mathcal{V}_{12}, \\
\frac{1}{2}V^{(2)\sigma}_{22} &= -\Delta + (\lambda_3+\lambda_4)\sigma_1^2 + 3\lambda_6\sigma_1\sigma_2 + 6\lambda_2\sigma_2^2 + 2\lambda_2\rho^2 \equiv \frac{1}{2}\mathcal{V}_{22}, \\
V^{(2)\sigma\pi}_{10} &= \Big(4(\lambda_3-\lambda_4)\sigma_1 + 2\lambda_6\sigma_2\Big)\rho \equiv \mathcal{V}_{10}\rho, \\
V^{(2)\sigma\pi}_{20} &= \Big(2\lambda_6\sigma_1 + 8\lambda_2\sigma_2\Big)\rho \equiv \mathcal{V}_{20}\rho, \\
\frac{1}{2}V^{(2)\pi}_{00} &= 4\lambda_2\rho^2 \equiv \frac{1}{2}\mathcal{V}_{00}\rho^2; \qquad \frac{1}{2}V^{(2)\pi}_{\pm\mp} = 0,
\end{aligned}
\tag{77}
$$

Now we are able to check the character of phase transition. Using consistently equations (16), (44) and the condition (69) in the SPB phase one obtains the differential equations on functions $\sigma_j(\mu),\ \rho(\mu)$, following the same strategy as for (74),

$$
\begin{aligned}
&\partial_\mu\sigma_1 = -4\mathcal{N}\sigma_1\sqrt{\mu^2-\sigma_1^2}\,\frac{\mathcal{V}_{22}\mathcal{V}_{00}-\mathcal{V}_{20}^2}{\mathrm{Det}\hat{\mathcal{V}}} < 0, \\
&\partial_\mu\sigma_2 = -4\mathcal{N}\sigma_1\sqrt{\mu^2-\sigma_1^2}\,\frac{\mathcal{V}_{10}\mathcal{V}_{20}-\mathcal{V}_{12}\mathcal{V}_{00}}{\mathrm{Det}\hat{\mathcal{V}}}, \\
&\partial_\mu\rho = -4\mathcal{N}\sigma_1\sqrt{\mu^2-\sigma_1^2}\,\frac{\mathcal{V}_{12}\mathcal{V}_{20}-\mathcal{V}_{10}\mathcal{V}_{22}}{\mathrm{Det}\hat{\mathcal{V}}}.
\end{aligned}
\tag{78}
$$

The last derivative must be positive in order to generate parity breaking and this is guaranteed by the inequality (76).

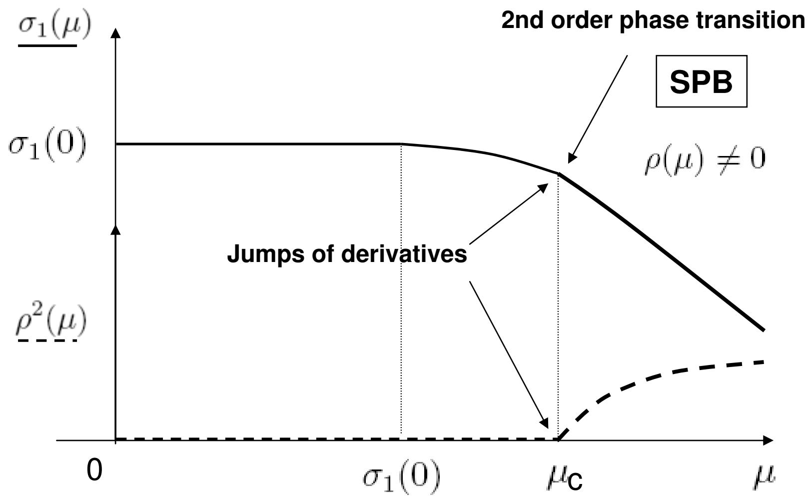

FIG. 2: *The SPB phase transition of second order: the dashed line depicts the SPB breaking phase and the solid line stands for the v.e.v. of "dynamical" mass. The plot is only qualitative*

Let us compare the derivatives of the dynamic mass σ_1 across the phase transition point. For $\mu \to \mu_{crit} - i0$ its derivative is given by (74) and for $\mu \to \mu_{crit} + i0$ it is given by (78).

Their difference reads

$$\partial_\mu \sigma_1\Big|_{\mu_{crit}+i0} - \partial_\mu \sigma_1\Big|_{\mu_{crit}-i0} = -4\mathcal{N}\sigma_1\sqrt{\mu^2-\sigma_1^2}\left\{\frac{\mathcal{V}_{22}\mathcal{V}_{00}-\mathcal{V}_{20}^2}{\mathrm{Det}\hat{\mathcal{V}}} - \frac{V_{22}^{(2)\sigma}}{\mathrm{Det}\hat{V}^{(2)\sigma}}\right\}$$
$$= -4\mathcal{N}\sigma_1\sqrt{\mu^2-\sigma_1^2}\,\frac{\left(\mathcal{V}_{10}V_{22}^{(2)\sigma} - \mathcal{V}_{20}V_{12}^{(2)\sigma}\right)^2}{\mathrm{Det}\hat{\mathcal{V}}\,\mathrm{Det}\hat{V}^{(2)\sigma}} < 0, \tag{79}$$

provided that both determinants are positive (they determine the spectrum of meson masses squared) and inequality (76) holds across. Thus the dynamic mass derivative is discontinuous and the phase transition is of second order.

B. Inclusion of current quark masses

For non-vanishing current quark masses the deviation linear in m_q in the parity breaking phase affects also the pseudoscalar parameters ρ and $\langle\pi^0\rangle$. After usage of equation (70) one finds

$$\hat{\Delta} \equiv \begin{pmatrix} \Delta\sigma_1(m_q) \\ \Delta\sigma_2(m_q) \\ \Delta\rho(m_q) \end{pmatrix} \simeq 2m_q \begin{pmatrix} \Delta_1 \\ \Delta_2 \\ \frac{\Delta_0}{\rho} \end{pmatrix}; \tag{80}$$
$$\tilde{\Delta} \equiv \begin{pmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_0 \end{pmatrix} = \left(\hat{\mathcal{V}}\right)^{-1}\cdot\tilde{d};\quad \tilde{d} \equiv \begin{pmatrix} d_1 \\ d_2 \\ \frac{d_2^2}{d_1\sigma_1+d_2\sigma_2} \end{pmatrix}\frac{d_1\sigma_1+d_2\sigma_2}{\sqrt{d_2^2\rho^2+(d_1\sigma_1+d_2\sigma_2)^2}},$$

where in order to keep the leading order all parameters must be taken in the chiral limit. As to the v.e.v. of neutral pion field it does not need any mass corrections to the leading order and must be taken from the

mass independent eq. (17)

$$\langle\pi^0\rangle = -\arctan\left(\frac{d_2\rho}{d_1\sigma_1 + d_2\sigma_2}\right). \tag{81}$$

Accordingly, the mass corrections to the matrix of second variation $\Delta\hat{V}^{(2)}$, equation (77), takes the form

$$V^{(2)\sigma}_{jl}(m_q) = V^{(2)\sigma}_{jl}(0) + \sum_{a=1,2,0}\partial_a\left(V^{(2)\sigma}_{jl}\right)\hat{\Delta}_a, \quad (\partial_a) \equiv (\partial_{\sigma_1}, \partial_{\sigma_2}, \partial_\rho)\,; \tag{82}$$

$$V^{(2)}_{\sigma_j\pi_2^0}(m_q) = V^{(2)\sigma\pi}_{j0} + \sum_{a=1,2,0}\partial_a\left(V^{(2)\sigma\pi}_{j0}\right)\hat{\Delta}_a, \tag{83}$$

$$V^{(2)}_{\sigma_1\pi_1^0} = \frac{2m_q d_1}{F_0}\sin\frac{\langle\pi^0\rangle}{F_0}, \quad V^{(2)}_{\sigma_2\pi_1^0} = \frac{2m_q d_2}{F_0}\sin\frac{\langle\pi^0\rangle}{F_0}, \tag{84}$$

$$\frac{1}{2}V^{(2)\pi^0}_{11} = m_q\left(\frac{d_1\sigma_1 + d_2\sigma_2}{F_0^2}\cos\frac{\langle\pi^0\rangle}{F_0} - \frac{d_2\rho}{F_0^2}\sin\frac{\langle\pi^0\rangle}{F_0}\right), \tag{85}$$

$$V^{(2)\pi^0}_{12} = \frac{2m_q d_2}{F_0}\cos\frac{\langle\pi^0\rangle}{F_0}, \tag{86}$$

$$\frac{1}{2}V^{(2)\pi^0}_{22} = 4\lambda_2\rho^2(0) + 16m_q\lambda_2\Delta_0 + \frac{m_q d_2^2}{d_1\sigma_1 + d_2\sigma_2}\cos\frac{\langle\pi^0\rangle}{F_0}, \tag{87}$$

$$V^{(2)}_{\pi_1^+\pi_1^-} = -2m_q\frac{d_2\rho}{\langle\pi^0\rangle^2}\sin\frac{\langle\pi^0\rangle}{F_0} = 2m_q\frac{d_1\sigma_1 + d_2\sigma_2}{\langle\pi^0\rangle^2}\frac{\sin^2\frac{\langle\pi^0\rangle}{F_0}}{\cos\frac{\langle\pi^0\rangle}{F_0}}, \tag{88}$$

$$V^{(2)}_{\pi_1^+\pi_2^-} = V^{(2)}_{\pi_2^+\pi_1^-} = \frac{2m_q d_2}{\langle\pi^0\rangle}\sin\frac{\langle\pi^0\rangle}{F_0}, \tag{89}$$

$$V^{(2)}_{\pi_2^+\pi_2^-} = \frac{2m_q d_2^2}{d_1\sigma_1 + d_2\sigma_2}\cos\frac{\langle\pi^0\rangle}{F_0}, \tag{90}$$

where the r.h.s. are evaluated with the help of eqs. (15)-(17), (69) - (71) and the v.e.v's for σ_j, ρ are taken in the chiral limit. We notice that convexity around this minimum implies that all diagonal elements are non-negative. This gives positive masses for two scalar and four pseudoscalar mesons, whereas the doublet of charged of π mesons remains massless. The latter can be easily checked from the vanishing determinant of the last matrix $V^{(2)}_{\pi_j^+\pi_l^-}$ in eqs. (88)-(90). Of course, quantitatively the mass spectrum can be obtained only after kinetic terms are properly normalized.

If the soft breaking of chiral symmetry occurs only in the H_1 channel, $d_2 = 0$ then it follows from eq. (17) that light pions don't condense $\langle\pi^0\rangle$ and don't mix with other states as the off-diagonal matrix elements (84), (86) and (89) vanish. The second pair of charged pseudoscalars π_2^- becomes massless manifestly.

VIII. KINETIC TERMS

In this section we examine the fluctuations around the constant solutions of the mass-gap equations (44),(16), (17),(18), and introduce appropriate notations for the fluctuations $\Sigma_j, \hat{\Pi}$ around v.e.v.'s $\bar{\sigma}_j, \rho$ so that $\sigma_j \equiv \bar{\sigma}_j + \Sigma_j$, $\hat{\pi}_2 = \tau_3\rho + \hat{\Pi}$. These v.e.v's $\bar{\sigma}_j, \rho$ must be used in all previous relations for the second variation of the potential. In calculations of the kinetic term we retain only the terms in the chiral limit keeping our interest to the masses of scalar and pseudoscalar mesons at the leading order of the expansion in current quark masses, m_q. Thus in the kinetic terms we take $\langle\pi_1\rangle \simeq 0$ according to eqs. (17),(18).

A. General form from chiral symmetry

Once we have fixed the interaction to quark matter we are not free in the choice of the kinetic term for scalar fields. Namely one cannot rotate two fields and rescale the field H_1 without changes in the chemical potential driver (55). However the rescaling of the field H_2 is possible at the expense of an appropriate redefinitions of other coupling constants and this freedom can be used to fix one of the constants which

appear in the kinetic term. Thus we take the general kinetic term symmetric under $SU(2)_L \times SU(2)_R$ global rotations to be

$$\mathcal{L}_{kin} = \frac{1}{4} \sum_{j,k=1}^{2} A_{jk} \mathrm{tr} \left\{ \partial_\mu H_j^\dagger \partial^\mu H_k \right\}. \tag{91}$$

With the chiral parameterization (9) one can separate the bare Goldstone boson action,

$$\begin{aligned}
\mathcal{L}_{kin} &= \frac{1}{2} \sum_{j,k=1}^{2} A_{jk} \partial_\mu \sigma_j \partial^\mu \sigma_k + \frac{1}{4} \sum_{j,k=1}^{2} A_{jk} \sigma_j \sigma_k \mathrm{tr} \left\{ \partial_\mu U^\dagger \partial^\mu U \right\} \qquad (92)\\
&+ \frac{1}{2} i \sum_{j=1}^{2} A_{j2} \mathrm{tr} \left\{ \sigma_j \left(\xi^\dagger (\partial_\mu \xi)^2 \xi^\dagger - \partial_\mu \xi (\xi^\dagger)^2 \partial^\mu \xi \right) \hat{\pi}_2 - \sigma_j \xi^\dagger \partial_\mu U \xi^\dagger \partial^\mu \hat{\pi}_2 + \partial_\mu \sigma_j \xi^\dagger \partial^\mu U \xi^\dagger \hat{\pi}_2 \right\} \\
&+ \frac{1}{4} A_{22} \mathrm{tr} \left\{ \partial_\mu \hat{\pi}_2 \partial^\mu \hat{\pi}_2 - 2 \partial_\mu \xi \xi^\dagger \hat{\pi}_2 \xi^\dagger \partial_\mu \xi \hat{\pi}_2 \right. \\
&\left. - (\partial_\mu \xi \xi^\dagger \partial^\mu \xi \xi^\dagger + \xi^\dagger \partial_\mu \xi \xi^\dagger \partial^\mu \xi)(\hat{\pi}_2)^2 + [\xi^\dagger, \partial_\mu \xi][\hat{\pi}_2, \partial^\mu \hat{\pi}_2] \right\}.
\end{aligned}$$

After selecting out the v.e.v. $\langle H_j \rangle = \langle \sigma_j \rangle \equiv \bar{\sigma}_j$ let us explore the kinetic part quadratic in fields. We expand $U = 1 + i\hat{\pi}_1/F_0 + \cdots$, $\xi = 1 + i\hat{\pi}_1/2F_0 + \cdots$ and use the notations defined at the beginning of this Section. Then the quadratic part looks as follows

$$\begin{aligned}
\mathcal{L}_{kin}^{(2)} &= \frac{1}{2} \sum_{j,k=1}^{2} A_{jk} \left[\partial_\mu \Sigma_j \partial^\mu \Sigma_k + \frac{1}{F_0^2} \bar{\sigma}_j \bar{\sigma}_k \partial_\mu \pi_1^a \partial^\mu \pi_1^a \right] \qquad (93)\\
&+ \frac{1}{F_0} \sum_{j=1}^{2} A_{j2} \left[-\rho \partial_\mu \Sigma_j \partial^\mu \pi_1^0 + \bar{\sigma}_j \partial_\mu \pi_1^a \partial^\mu \Pi^a \right] + \frac{1}{2} A_{22} \left[\frac{\rho^2}{F_0^2} \partial_\mu \pi_1^0 \partial^\mu \pi_1^0 + \partial_\mu \Pi^a \partial^\mu \Pi^a \right],
\end{aligned}$$

which shows manifestly the mixture between light and heavy pseudoscalar states and, in the SPB phase, also between scalar and pseudoscalar states.

Let us define

$$F_0^2 = \sum_{j,k=1}^{2} A_{jk} \bar{\sigma}_j \bar{\sigma}_k, \quad \zeta \equiv \frac{1}{F_0} \sum_{j=1}^{2} A_{j2} \bar{\sigma}_j. \tag{94}$$

B. Parity-symmetric phase

In the symmetric phase $\rho = 0$, $\hat{\pi}_2 = \hat{\Pi}$ one diagonalizes by shifting the pion field

$$\pi_1^a = \tilde{\pi}^a - \zeta \pi_2^a, \tag{95}$$

$$\mathcal{L}_{kin,\pi}^{(2)} = \frac{1}{2} \partial_\mu \tilde{\pi}^a \partial^\mu \tilde{\pi}^a + \frac{1}{2} (A_{22} - \zeta^2) \partial_\mu \pi_2^a \partial^\mu \pi_2^a, \qquad A_{22} - \zeta^2 = \frac{\bar{\sigma}_1^2 \det A}{F_0^2} > 0. \tag{96}$$

Taking into account the modification of the matrix of second variations (85)-(90) after shifting (95) one finds the masses of light and heavy pseudoscalars to the leading order in current quark mass

$$\begin{aligned}
(\tilde{V}_{11}^{(2)\pi})^{ab} &= (V_{11}^{(2)\pi})^{ab} = \delta^{ab} 2 m_q \frac{d_1 \bar{\sigma}_1 + d_2 \bar{\sigma}_2}{F_0^2} = \delta^{ab} m_\pi^2, \\
(\tilde{V}_{12}^{(2)\pi})^{ab} &= (V_{12}^{(2)\pi})^{ab} - \zeta (V_{11}^{(2)\pi})^{ab} = \delta^{ab} 2 m_q \left(\frac{d_2}{F_0} - \zeta \frac{d_1 \bar{\sigma}_1 + d_2 \bar{\sigma}_2}{F_0^2} \right), \\
(\tilde{V}_{22}^{(2)\pi})^{ab} &= \left((V_{22}^{(2)\pi})^{ab} - 2\zeta (V_{12}^{(2)\pi})^{ab} + \zeta^2 (V_{11}^{(2)\pi})^{ab} \right) \\
&= \delta^{ab} 2 \Big(-\Delta + (\lambda_3 - \lambda_4) \bar{\sigma}_1^2 + \lambda_6 \bar{\sigma}_1 \bar{\sigma}_2 + 2\lambda_2 \bar{\sigma}_2^2 - \zeta m_q \frac{2 d_2}{F_0} + \zeta^2 m_q \frac{d_1 \bar{\sigma}_1 + d_2 \bar{\sigma}_2}{F_0^2} \Big) \\
&= \delta^{ab} ((A_{22} - \zeta^2)) m_\Pi^2 + \mathcal{O}(m_q^2). \qquad (97)
\end{aligned}$$

at the leading order in m_q when $m_\Pi \gg m_\pi$ far below the P-breaking transition point in chemical potential. We notice that in this region the off-diagonal element does not make any influence.

At the point of the SPB phase transition one has to impose the condition (69) which leads to

$$(\tilde{V}_{22}^{(2)\pi})^{ab} = \delta^{ab} \frac{2m_q}{d_1\bar{\sigma}_1 + d_2\bar{\sigma}_2}\Big(d_2 - \zeta\frac{d_1\bar{\sigma}_1 + d_2\bar{\sigma}_2}{F_0}\Big)^2. \tag{98}$$

Evidently the determinant of $(\tilde{V}^{(2)\pi})$ vanishes and one reveals zero modes for all three pion states, one neutral and two charged. They represent the true Goldstone modes (in the limit of exact isospin symmetry $m_u = m_d$). At the P-breaking transition point $\rho = 0$, when taking into account the normalization of kinetic terms (93) with the definitions (94) one finds the values of three massive modes

$$m_{\tilde{\Pi}}^2 = 2m_q \frac{A_{11}d_2^2 - 2A_{12}d_1d_2 + A_{22}d_1^2}{\det A(d_1\bar{\sigma}_1 + d_2\bar{\sigma}_2)} = 2m_q \frac{(\vec{d}A^{-1}\vec{d})}{(\vec{d}\,\vec{\bar{\sigma}})}. \tag{99}$$

Thus in the chiral limit, at the phase transition point one reveals six zero modes and beyond the chiral limit only three ones.

C. Masses of light states in SPB phase

In the SPB phase the situation is more involved: pseudoscalar states mix with scalar ones. In particular, the diagonalization of kinetic terms is different for neutral and charged pions because the vector isospin symmetry is broken: $SU(2)_V \to U(1)$. Namely

$$\tilde{\pi}^\pm = \pi^\pm + \zeta\Pi^\pm, \qquad \tilde{\pi}^0 = \pi^0 + \frac{F_0^2}{F_0^2 + A_{22}\rho^2}\Big(\zeta\Pi^0 - \frac{\rho}{F_0}\sum_{j=1}^{2} A_{j2}\Sigma_j\Big). \tag{100}$$

In this way SPB induces mixing of both massless and heavy neutral pions with scalars. The (partially) diagonalized kinetic term has the following form

$$\begin{aligned}
\mathcal{L}_{kin}^{(2)} &= \partial_\mu\tilde{\pi}^\pm\partial^\mu\tilde{\pi}^\mp + \frac{1}{2}\Big(1 + \frac{A_{22}\rho^2}{F_0^2}\Big)\partial_\mu\tilde{\pi}^0\partial^\mu\tilde{\pi}^0 + (A_{22} - \zeta^2)\partial_\mu\Pi^\pm\partial^\mu\Pi^\mp \\
&+ \frac{1}{2}(A_{22} - \frac{F_0^2}{F_0^2 + A_{22}\rho^2}\zeta^2)\partial_\mu\Pi^0\partial^\mu\Pi^0 \\
&+ \frac{1}{2}\sum_{j,k=1}^{2}\frac{A_{jk}F_0^2 + \rho^2\det A\delta_{1j}\delta_{1k}}{F_0^2 + A_{22}\rho^2}\partial_\mu\Sigma_j\partial^\mu\Sigma_k - \frac{F_0\rho}{F_0^2 + A_{22}\rho^2}\zeta\partial_\mu\Pi^0\sum_{j=1}^{2}A_{j2}\partial^\mu\Sigma_j.
\end{aligned} \tag{101}$$

We see that even in the massless pion sector the isospin breaking $SU(2)_V \to U(1)$ occurs: neutral pions become less stable with a larger decay constant. Another observation is that in the charged meson sector the relationship between massless π and π' remain the same as in the symmetric phase.

Beyond the chiral limit one can derive the masses of the lightest pseudo-goldstone states. When $\rho \gg m_q$ then in the mass matrix (82)- (90) the heavy mass parts (82), (83), (87) and the light mass ones (85), (88) - (90) combine into an approximately block diagonal form with additional off-diagonal elements (84) and (86), proportional to m_q. The latter leads to factorization of the light pseudoscalar meson sector from the heavy meson one to the order of m_q^2. Thus neglecting the mixture of heavy and light states one deals with the light sector built of (85), (88) -(90) which after diagonalizing the kinetic term by (100) projected on the light state sector gives the light pseudoscalar masses

$$\begin{aligned}
m_{\tilde{\pi}_0}^2 &= 2m_q\Big(1 + \frac{A_{22}\rho^2}{F_0^2}\Big)^{-1}\left(\frac{d_1\bar{\sigma}_1 + d_2\bar{\sigma}_2}{F_0^2}\cos\frac{\langle\pi^0\rangle}{F_0} - \frac{d_2\rho}{F_0^2}\sin\frac{\langle\pi^0\rangle}{F_0}\right), \\
m_{\tilde{\pi}^\pm}^2 &= 0,
\end{aligned} \tag{102}$$

$$m_{\Pi^\pm}^2 = 2m_q\frac{\cos\frac{\langle\pi^0\rangle}{F_0}}{(A_{22} - \zeta^2)(d_1\bar{\sigma}_1 + d_2\bar{\sigma}_2)}\left(d_2 - \zeta\frac{d_1\bar{\sigma}_1 + d_2\bar{\sigma}_2}{\langle\pi^0\rangle}\tan\frac{\langle\pi^0\rangle}{F_0}\right)^2. \tag{103}$$

Thus in the SPB one finds two massless charged pseudoscalars and three light pseudoscalars with masses

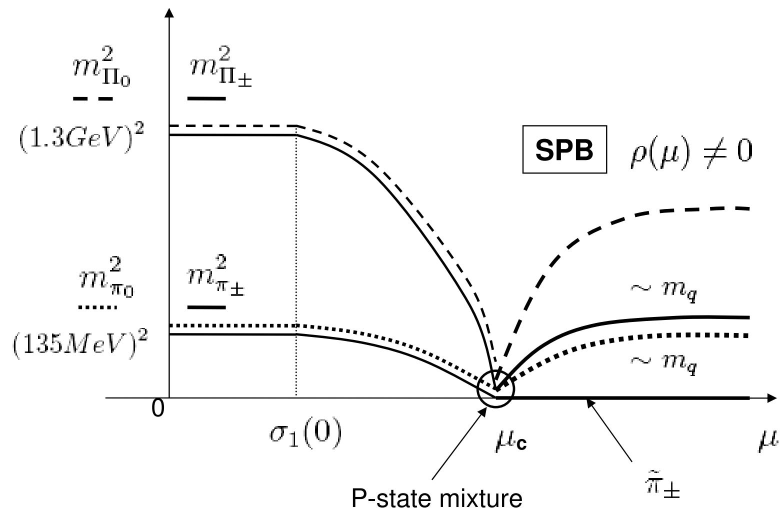

FIG. 3: *Masses of light states in SPB phase: the vacuum masses are taken from [40]. The light and heavy charged pseudoscalars are depicted with solid lines, the dotted line corresponds to the neutral light pseudoscalar and the dashed line stands for the neutral heavy one. The plot is only qualitative.*

linear in the current quark mass. These equations represent the generalization of the Gell-Mann-Oakes-Renner relation in the phase with broken parity.

We notice that the masses of neutral and charged pseudoscalars do not coincide in the well developed SPB phase, just realizing the spontaneous breaking of isospin symmetry. One can also guess that the manifest breaking of $SU(2)$ symmetry due to different masses of u and d quarks will supply the Goldstone bosons $\tilde{\pi}^{\pm}$ with tiny masses proportional to the difference $m_u - m_d$.

IX. DESCRIPTION OF NUCLEAR MATTER

The normal density of infinite nuclear matter [26] is $\varrho_0 \simeq 0.15 \div 0.16$ fm^{-3} that corresponds to the average distance $1.8 \div 1.9$ fm between nucleons in nuclear matter. We assume that the quark matter is equivalent to nuclear matter when their average baryon densities coincide, at least in what respects meson properties. This normalization we will apply at the normal baryon density while noting that there also exist different proposals [41] to confront quark and nuclear matter for higher densities.

Thermodynamical characteristics are the pressure, P, and the energy density, ε. The pressure is determined in the presence of chemical potential by (49), defined for σ_j satisfying the mass gap equation. Evidently the pressure at zero nuclear density vanishes. In this case the energy and baryon densities are related to the pressure as follows

$$\varepsilon = -P + N_c \mu \varrho_B; \ \partial_\mu P = N_c \varrho_B; \ \partial_\varrho \varepsilon = N_c \mu. \tag{104}$$

The direct connection between energy density and pressure reads

$$P = \varrho_B^2 \partial_\varrho \left(\frac{\varepsilon}{\varrho_B} \right). \tag{105}$$

Evidently the energy per baryon has an extremum when the pressure vanishes. Since the pressure is an increasing function of the density as we have seen, obviously vanishing at zero density, and infinite nuclear matter is stable (thus implying zero pressure) the phase diagram in the P, ϱ_B plane is necessarily discontinuous with values of density in the interval $(0, \varrho_0)$ not corresponding to equilibrium states (ϱ_0 is nuclear matter density). We will see below how this is realized in our model.

A. On the way to stable nuclear matter

Our model consisting of two scalar isomultiplets is still somewhat too simple in one aspect. The stabilization of nuclear matter requires not only attractive scalar forces (scalars) but also repulsive ones (vector-mediated). Conventionally [26], the latter ones are associated to the interactions mediated by the iso-singlet

vector ω meson. Let us supplement our action with the free ω meson lagrangian and its coupling to quarks

$$\Delta\mathcal{L}_\omega = -\frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - g_{\omega\bar{q}q}\bar{q}\gamma_\mu\omega^\mu q, \tag{106}$$

with a coupling constant $g_{\omega\bar{q}q} \sim \mathcal{O}(1/\sqrt{N_c})$. After bosonization of QCD, on symmetry grounds, any vector field interacts with scalars in the form of commutator and therefore ω_μ does not show up in the effective potential H_j fields to the lowest order. However in the quark lagrangian the time component ω_0 interplays with the chemical potential and it is of importance to describe the dense nuclear matter properties. Let us assign a constant v.e.v. for this component $g_{\omega\bar{q}q}\langle\omega_0\rangle \equiv \tilde{\omega}$. Then one needs to compute the modification of the effective potential due to the replacement $\mu \to \mu + \tilde{\omega} \equiv \tilde{\mu}$. The variable $\tilde{\omega}$, and accordingly $\tilde{\mu}$, is dynamical and in addition appears quadratically in the mass term in (106) which reads

$$\Delta V_\omega = -\frac{1}{2}m_\omega^2\langle\omega_0^2\rangle = -\frac{1}{2}\frac{(\tilde{\mu}-\mu)^2}{G_\omega}, \quad G_\omega \equiv \frac{g^2_{\omega\bar{q}q}}{m_\omega^2} \simeq \mathcal{O}(\frac{1}{N_c}). \tag{107}$$

The term (107) supplements the effective potential (56): $\widetilde{V}_{\text{eff}}(\mu) = V_{\text{eff}}(\tilde{\mu}) + \Delta V_\omega(\mu, \tilde{\mu})$. Correspondingly the extremum condition for the variation of the variable $\tilde{\mu}$ involves both the scalar part of the effective potential (56) and the vector one (107) and due to (52) takes the following form

$$N_c\varrho_B(\mu) = \frac{N_cN_f}{3\pi^2}p_F^3(\tilde{\mu}) = \frac{\mu - \tilde{\mu}}{G_\omega}, \tag{108}$$

from this one finds $\tilde{\mu}(\mu)$ after solving the mass-gap equations (44), (16) and (18).

Finally the extended effective potential at a minimum reads

$$\begin{aligned}\widetilde{V}_{\text{eff}}(\mu) &= -\frac{1}{2}\Delta\Big(\sigma_1^2(\tilde{\mu}) + \sigma_2^2(\tilde{\mu}) + \rho^2(\tilde{\mu})\Big) \\ &\quad -\Big[\frac{\mathcal{N}}{3}\tilde{\mu}\Big(\tilde{\mu}^2 - \sigma_1(\tilde{\mu})\Big)^{3/2} + G_\omega\frac{8\mathcal{N}^2}{9}\Big(\tilde{\mu}^2 - \sigma_1(\tilde{\mu})\Big)^3\Big]\theta\Big(\tilde{\mu} - \sigma_1(\tilde{\mu})\Big).\end{aligned} \tag{109}$$

Let us define the v.e.v.'s of scalar field σ_1 in vacuum at the two minima as $\sigma_1^*(0) < \sigma_1^\sharp(0)$. Let us select out the parameter subspace such that the minimum corresponding to $\sigma_1^\sharp(0), \sigma_2^\sharp(0)$ is lower than the another minimum at $\sigma_1^*(0), \sigma_2^*(0)$. Then for parity-even matter $\rho = 0$, one seeks for the nuclear matter stability at a value of chemical potential $\tilde{\mu}_s$ with $\sigma_1^*(0) \le \tilde{\mu}_s < \sigma_1^\sharp(0)$. The corresponding baryon matter stability condition $\Delta P = P(\sigma_1^\sharp(0)) - P(\sigma_1^*(\tilde{\mu}_s)) = 0$, eq. (49), can be formulated as

$$\begin{aligned}\Delta\Big((\sigma_1^\sharp(0))^2 + (\sigma_2^\sharp(0))^2 - (\sigma_1^*(\tilde{\mu}_s))^2 - (\sigma_2^*(\tilde{\mu}_s))^2\Big) &= \frac{N_cN_f}{6\pi^2}\tilde{\mu}_s p_F^3(\tilde{\mu}_s) + G_\omega\frac{N_c^2N_f^2}{9\pi^4}p_F^6(\tilde{\mu}_s) \\ &= \frac{N_c}{2}\tilde{\mu}_s\varrho_B(\mu_s) + G_\omega N_c^2\varrho_B^2(\mu_s),\end{aligned} \tag{110}$$

taking into account (109) and (107) . Herein $\tilde{\mu}_s$ is related to the physical value of μ_s by (108) and it is assumed that parity is not violated $\rho = 0$. This relation represents the condition for the formation of stable symmetric nuclear matter in result of first-order phase transition [26]. It can be fulfilled by an appropriate choice of the vector coupling constant G_ω as typically the first term in the r.h.s. of (110) is smaller than the one on the l.h.s.

At finite temperatures one has to modify the thermodynamic relations. The modification of the effective potential due to ω mesons is given by (107). Thus $\tilde{V}_{\text{eff}}(\mu, \beta) \equiv V_{\text{eff}}(\tilde{\mu}, \beta) + \Delta V_\omega(\tilde{\mu}, \mu)$ which should henceforth be used in all the previous thermodynamical formulae. The replacement $\mu \to \tilde{\mu}$ makes all expectation values depend rather on $\tilde{\mu}$ which is determined via the variation of $\tilde{V}_{\text{eff}}$

$$\frac{\tilde{\mu} - \mu}{G_\omega} = -N_c\varrho_B\Big(\beta, \mu, \sigma_1\Big) = \partial_{\tilde{\mu}}\Delta V_{\text{eff}}(\tilde{\mu}, \beta). \tag{111}$$

The saturation point at $\mu = \mu_s$ where nuclear matter forms is characterized by the energy crossing condition for $P, T \neq 0$,

$$\begin{aligned}\Delta\Big(\big(\sigma_1^\sharp\big)^2 + \big(\sigma_2^\sharp\big)^2 - \big(\sigma_1^*\big)^2 - \big(\sigma_2^*\big)^2\Big) &= \frac{N_c}{2}\tilde{\mu}_s\Big(\varrho_B(\beta, \mu_s, \sigma_1^*) - \varrho_B(\beta, \mu_s, \sigma_1^\sharp)\Big) \\ +\frac{1}{2}T\Big(s(\beta, \mu_s, \sigma_1^*) - s(\beta, \mu_s, \sigma_1^\sharp)\Big) &+ G_\omega N_c^2\Big(\varrho_B^2(\beta, \mu_s, \sigma_1^*) - \varrho_B^2(\beta, \mu_s, \sigma_1^\sharp)\Big),\end{aligned} \tag{112}$$

where $\tilde{\mu}_s$ is related to the physical value of μ_s by equation (111) and $\sigma_j^\sharp \equiv \sigma_j^\sharp(\tilde{\mu}_s, \beta)$; $\sigma_j^* \equiv \sigma_j^*(\tilde{\mu}_s, \beta)$. This relation represents the condition for the existence of symmetric nuclear matter. It can be always fulfilled by an appropriate choice of G_ω. It can be shown that normal nuclear matter is formed at the chemical potential $\mu_s \simeq 303MeV$ stabilized by ω meson condensate with $G_\omega \sim (10 \div 15)GeV^{-2}$ in a satisfactory agreement to what is known from other model estimations [42].

B. First-order transition without chiral collapse

A viable model of dense baryon (quark/nucleon) matter must reveal the phase transition to a stable bound state at the normal nuclear density $\varrho_B = \varrho_0$ for infinite homogeneous symmetric nuclear matter at the so called "saturation point". This phase transition is believed to be of first order similar to the vapor condensation into liquid: from droplets of heavy nuclei to a homogeneous nuclear liquid. However in simple quark models of the NJL type [38] this phase transition (for vanishing current quark masses) goes to the chirally symmetric phase with zero dynamical mass (zero v.e.v. of scalar fields), so called "chiral collapse". When it happens the typical baryon density is substantially larger that the normal one $\varrho_{B,c} = 2.8\varrho_0$. For this reason these simple models cannot be a reasonable guide to phase transitions in very dense nuclear matter.

Let us examine under which conditions the saturation point in our model happens to be at normal nuclear density and is not accompanied by the chiral collapse keeping the dynamical mass different from zero. To analyze this problem we have to examine the pressure in cold ($T = 0$), dense baryon matter.

We remind that in so far as our system undergoes spontaneous CSB the effective potential (32) does not reveal any minimum at the origin in variables σ_j (for $\mu = 0$) and may have either a saddle point, $\det\left[\hat{V}^{(2)}\right](\sigma_j = 0) < 0$ or a maximum, $\det\left[\hat{V}^{(2)}\right](0) > 0$, $\mathrm{tr}\left\{\hat{V}^{(2)}\right\}(0) < 0$. One has a positive definite matrix of second variations (21), (22) of the effective potential in the vicinity of a CSB solution, $\det\left[\hat{V}^{(2)}\right](\sigma_j^\sharp) > 0$; $\mathrm{tr}\left\{\hat{V}^{(2)}\right\}(\sigma_j^\sharp) > 0$. It means that $V_{\text{eff}}(\sigma_j^\sharp) < V_{\text{eff}}(0)$.

These properties allow us to guess that at some value of chemical potential $\mu_s < \sigma_1^\sharp$ and smaller values of v.e.v.'s for scalar fields $\sigma_1^* < \mu_s$ the deficit in scalar background energies on the left-hand side of (110) may be exactly compensated by contributions from the nuclear density and omega-meson repulsion on the right-hand side so that $P(\sigma_j^*, \mu_s) = 0$ and the system undergoes a first-order phase transition to the stable quark (nuclear) matter.

Let us now prove that for a large variety of coupling constants admitting CSB (and SPB, see below) one of the v.e.v. $\sigma_j^* \neq 0$ in the chiral limit, and the chiral collapse is impossible. Indeed, suppose that $\sigma_j^* = 0$ at $\mu^* < \sigma_1^\sharp$ where the pressure vanishes then

$$(\mu^*)^4 + \frac{8G_\omega\mathcal{N}}{3}(\mu^*)^6 = -\frac{3}{\mathcal{N}}\left|V_{\text{eff}}\,(\sigma_j^\sharp, \mu = 0)\right|. \tag{113}$$

In this case the second variation matrix for the effective potential (21), (58) reads

$$\frac{1}{2}V_{11}^{(2)\sigma} = -\Delta + \mathcal{N}(\mu^*)^2,\ V_{12}^{(2)\sigma} = 0,\ \frac{1}{2}V_{22}^{(2)\sigma} = -\Delta. \tag{114}$$

In order to induce SPB one takes $\Delta > 0$ (see below). Then from (114) one finds that for any value of μ^* the second variation matrix is never positive definite and one reveals either a saddle point or a maximum at a presumed saturation point whereas a maximum remains for vacuum values with $\mu^* < \sigma_1^\sharp$. As we have to guarantee the existence of stable nuclear matter with normal baryon density we consider further on $\Delta > 0$.

It is instructive to reduce the two-multiplet sigma model to a one-multiplet lagrangian associated to a NJL quark model. In relations (114) it simply corresponds to taking only one matrix element $V_{11}^{(2)\sigma}$ to describe the behavior around extremum at the origin. Evidently, there is always a value of μ^* for which it becomes positive and chiral collapse is inevitable. But when comparing with our model one concludes that the reason for appearance of chiral collapse is not the absence of confinement [38] but the inefficiency of a one-channel linear sigma model in representing the complicated chiral dynamics in hadronic physics.

C. (In)compressibilities: matching quark and nuclear matters

The (in)compressibility in the quark matter must be defined as $K(\mu) = \partial_{\varrho_B} P$, where the derivative is made with the help of the function $\varrho_B(\mu)$ given in (59). For a zero pressure state (such as stable nuclear

matter) this equals

$$K = \varrho_B^2 \partial_{\varrho_B}^2 \left(\frac{\varepsilon}{\varrho_B} \right) |_{P=0}, \tag{115}$$

which must be positive since it corresponds to a minimum of the energy per baryon. In our model this is indeed the case

$$K(\mu) = N_c \varrho_B \partial_{\varrho_B} \mu = \frac{p_F^2(\tilde{\mu})}{\Big(\tilde{\mu} - \sigma_1 \partial_{\tilde{\mu}} \sigma_1(\tilde{\mu})\Big)} + 9 G_\omega \varrho_B(\mu), \quad \partial_{\tilde{\mu}} \sigma_1 = -4 \mathcal{N} \sigma_1 \sqrt{\tilde{\mu}^2 - \sigma_1^2}\, \frac{V^{(2)}_{\sigma_2 \sigma_2}}{\det \hat{V}^{(2)}} < 0. \tag{116}$$

where $N_c = 3$ is assumed.

In order to adjust the properties of stable baryon matter in a quark description we parameterize the (in)compressibility in the following way: $K = a N_c \varrho_B \partial_\varrho \mu \Big|_{P=0}$. Let us show that the matching of quark and nuclear matter at the saturation point is provided by the normalization factor $a = 9$ in meson-nucleon models and $a = 1$ for quark-meson models. The incompressibility must be positive (giving a minimum of the energy per baryon) when a stable nuclear matter is formed at zero pressure. The derivative of the dynamical mass is given in (116) wherefrom it follows that the derivative of the pressure is always positive. Thus if there is a solution with $\sigma_1^* \equiv \sigma_1(\mu^*) < \mu^* < \sigma_1^\sharp$ providing $P = 0$, the phase transition emerges to the stable nuclear matter state.

In the terms of nuclear d.o.f. one defines

$$\varepsilon = -P + \mu_N \varrho_B. \tag{117}$$

At the saturation point $P = 0$ and

$$\mu_N = \frac{\varepsilon}{\varrho_B} = \text{energy per baryon} = m_N - E_{bound} = (939 - 16) MeV = 923 MeV. \tag{118}$$

The quark matter chemical potential is defined as $\partial_\varrho \varepsilon = \mu_N = N_c \mu$. Therefore at the saturation point $\mu_s = 308 MeV$. Let us use this point for the quark-hadron matching

$$\varrho_B(\mu_N) = \varrho_B(\mu_s), \quad p_{F,N}^2 = (\mu_N)^2 - (m_N)^2 \simeq p_{F,q}^2 = (\mu_s)^2 - (\sigma_1^*)^2, \tag{119}$$

if we neglect the vector meson shift $\tilde{\mu} \simeq \mu$. However the matching of densities does not provide the equivalence of derivatives w.r.t. chemical potentials. Namely around the saturation point

$$\frac{\partial p_F^2}{\partial \mu_N} \simeq 2 \mu_N = 2 N_c \mu_s \neq \frac{\partial p_F^2}{\partial \mu} \simeq 2 \mu_s, \tag{120}$$

where (subdominant) derivatives of dynamical masses are neglected for a moment. Now let us try to extend the matching to the (in)compressibilities, providing the correct N_c factors. For hadron matter

$$K_N(\mu_s) = 9 \varrho_B \Big(\frac{\partial \varrho_B}{\partial \mu_N} \Big)^{-1} \simeq 3 \frac{p_F^2}{\mu_N} = \frac{3 p_F^2}{N_c \mu_s}, \tag{121}$$

whereas for quark matter,

$$K_Q(\mu_s) = N_c \varrho_B \Big(\frac{\partial \varrho_B}{\partial \mu} \Big)^{-1} \simeq \frac{N_c p_F^2}{3 \mu_s}, \tag{122}$$

they match each other for $N_c = 3$. One could do things even better. If the coefficient for hadron matter we redefine $9 \to 3N_c$ and we introduce the coefficient $3/N_c$ for quark matter then both definitions match for any N_c to the leading order. At least one then could succeed in their matching around normal density.

Going back to quark matter description it would mean that

$$K(\mu_s) = \frac{p_F^2(\tilde{\mu}_s)}{\Big(\tilde{\mu}_s - \sigma_1 \partial_{\tilde{\mu}} \sigma_1(\tilde{\mu}_s)\Big)} + 3 N_c G_\omega \varrho_B(\mu_s), \tag{123}$$

has a finite limit at large N_c coinciding with what we get for hadron matter if one remembers that $G_\omega \sim 1/N_c$.

D. Saturation point meets spontaneous parity breaking

Let us search for the domain of parameters in the model providing the realization of both stable nuclear matter and the regime of SPB. The former is associated with a first-order phase transition and implies the existence of two minima at zero chemical potential which are possibly moving when the chemical potential increases. The highest, metastable minimum must start moving at chemical potentials μ smaller than the value of the dynamical mass of the lowest minimum, $\sigma_1^\sharp$ and larger than the v.e.v. σ_1^* at the highest minimum, $\sigma_1^* \leq \mu < \sigma_1^\sharp$. This metastable minimum may reach the lowest one if the density and omega meson effects are taken into account. Then a first-order phase transition to normal nuclear matter occurs when pressures become equal, eq. (110).

In order to simplify our search we make a particular choice of $\lambda_5 = 0$ (not a reduction by (29)!). In this case one of the solutions is $\sigma_2^{(0)} = 0$ and $2\lambda_1(\sigma_1^{(0)})^2 = \Delta$ and it is a *minimum* as it follows from (21), (22) (in the chiral limit) provided that $\lambda_1 > 0, (\lambda_3 + \lambda_4) > 2\lambda_1$. When $\sigma_2^{(0)} = 0$ a higher symmetry $Z_2 \times Z_2$ arises for the effective potential in the vicinity of such a minimum as the contribution of the vertex with λ_6 into the second variation vanishes with $\sigma_2^{(0)}$. For $\sigma_2 \neq 0$ one can obtain eq. (35) for the ratio $t = \sigma_2/\sigma_1$. As it is analyzed in subsection 4.2, it has, in general, one or three real roots. For our purposes eq. (35) must have three real solutions: one corresponding to a minimum $t^{(3)} > 0$ and two corresponding to saddle points $t^{(1)} < 0, t^{(2)} > 0$. The inequality controlling the existence of three real solutions is derived in subsection 4.2 from the analysis of the minimum of the polynomial (35). Finally for a given solution $t^{(3)}$ one finds a unique solution for $\sigma_1 > 0$ from (15) (or (44)) and (16) .

Let us assume the minimum with $\sigma_2^{(0)} = 0$ to be the higher one at zero chemical potentials $\sigma_1^{(0)} \equiv \sigma_1^*$. For this choice to be realized it is *sufficient* to fulfill the inequality $\sigma_1^{(0)} \equiv \sigma_1^* < \sigma_1^{(3)} \equiv \sigma_1^\sharp$. It turns out that in order to provide it one has to satisfy the inequality

$$2\lambda_2 t^2 + \frac{3}{2}\lambda_6 t + (\lambda_3 + \lambda_4 - 2\lambda_1) < 0, \tag{124}$$

which implies

$$9\lambda_6^2 \geq 32\lambda_2(\lambda_3 + \lambda_4 - 2\lambda_1); \quad 0 < t^{(3)} < -\frac{3\lambda_6}{4\lambda_2}. \tag{125}$$

When at the critical value of $\mu = \mu_s < \sigma_1^\sharp$, the solution with $\sigma_2^* = 0$ describes a saturation point then the further evolution of the meson background for higher chemical potentials is characterized by the following equation $\sigma_1^*(\mu^*)$

$$2\lambda_1(\sigma_1^*)^2(\mu) = \Delta - \mathcal{N}\mathcal{A}(\sigma_1^*, \mu). \tag{126}$$

As the last term is monotonously increasing with chemical potential the v.e.v of scalar field is decreasing. Now we approach to the P-breaking regime and employ eq. (60) at the expected phase transition point. Its solution is

$$\sigma_{1,c}^2 = \frac{\Delta}{\lambda_3 - \lambda_4} < (\sigma_1^\#)^2 = \frac{\Delta}{2\lambda_1}. \tag{127}$$

Thus the feasibility of spontaneous P-breaking depends on the realization of the inequality $\sigma_{1,c} < \sigma_1^* < \sigma_1^\#$.

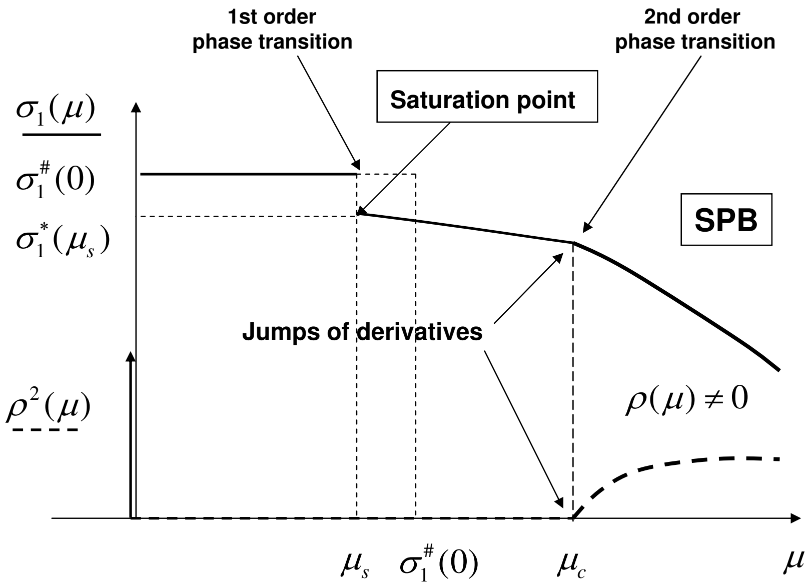

FIG. 4: *Saturation point meets spontaneous parity breaking: at $\mu = \mu_s$ the pressures for the two solutions $\sigma^\sharp, \sigma^*$ become equal and the solutions interchange realizing the 1st order phase transition. At a larger chemical potential μ_c the 2nd order SPB phase transition occurs.*

Let us collect the inequalities providing the required convexity of the two minima and the very existence of both the stable nuclear matter and a parity breaking phase for higher densities (see [18, 19])

$$-\frac{3\lambda_6}{4\lambda_2} > t^{(3)} > \max\Big[-\frac{3\lambda_6}{8\lambda_2}; -\frac{4\lambda_4}{\lambda_6}\Big],\ \lambda_{1,2,3,4} > 0,\ \lambda_3 > \lambda_4,\ \frac{3}{2}\lambda_6^2 > 8\lambda_2\lambda_4 > \lambda_6^2,$$
$$\Delta > 0,\ (\lambda_3 \pm \lambda_4) > 2\lambda_1,\ (\lambda_3 + \lambda_4) > 2\lambda_2, \tag{128}$$

in addition to those ones derived above. For a more definite numerical estimation of these six constants there is not at present enough experimental or phenomenological information, although it can be shown that the tentative values assumed in [18] for $\lambda_1 \sim 0.15, \lambda_3 \sim 4, \Delta \sim 0.03 GeV^{-2}$ may lead to the occurrence of SPB at about three times normal nuclear densities.

X. CONCLUSIONS

Let us summarize here our main findings. Parity breaking seems to be quite a realistic possibility in nuclear matter at moderate densities. We have arrived at this conclusion by using an effective lagrangian for low-energy QCD that retains the two lowest lying states in the scalar and pseudoscalar sectors. We include a chemical potential for the quarks that corresponds to a finite density of baryons and investigate the pattern of symmetry breaking in its presence. We have found the necessary and sufficient conditions for a phase where parity is spontaneously broken to exist. In general this phase is bound and it extends across a range of chemical potentials that correspond to nuclear densities where more exotic phenomena such as color-flavor locking or color superconductivity may occur. It also extends to finite temperature.

Salient characteristics of the phase with parity breaking would be the spontaneous breaking of the vector isospin symmetry $SU(2)_V$ down to $U(1)$ and the generation two additional massless charged pseudoscalar mesons. We also find a strong mixing between scalar and pseudoscalar states that translate spontaneous parity breaking into meson decays. The mass eigenstates will decay both in odd and even number of pions simultaneously. Isospin breaking can also be visible in decay constants.

Let us mention here several possible experimental or phenomenological signatures of parity breaking, discussed previously in [18, 19] in the chiral limit.

a) Decays of higher-mass meson resonances (radial excitations) into pions. Resonances do not have a definite parity and therefore the same resonance can decay both in two and three pions (in general into even and odd number of pions).

b) At the very point of the phase transition leading to parity breaking one has *six* massless pion-like states in the chiral limit and the *two* massless charged states when the quark masses are taken into account. After crossing the phase transition, in the parity broken phase, the massless charged pseudoscalar states remain as Goldstone bosons enhancing charged pion production, whereas the additional neutral pseudoscalar state becomes massive.

c) Reinforcement of long-range correlations in the pseudoscalar channel.

d) Additional isospin breaking effects in the pion decay constant and substantial modification of $F_{\pi'}$ for massless charged pions, giving an enhancement of electroweak decays.

We think that our conclusions are drawn in a region of parameters where our effective lagrangian is applicable and, while obviously we cannot claim high accuracy in our predictions, we are confident that the existence of this novel phase is not an spurious consequence of our approach but a rather robust prediction. It would surely be interesting to investigate how this new phenomenon could possibly influence the equation of state of neutron stars (the density of such objects seems to be about right for it).

One could hope that lattice methods could shed some light on this issue and confirm or falsify the existence of this interesting phase in dense nuclear matter. The problem, as discussed in the introduction, is far from trivial. One could probably use the expansions for small values of μ at finite temperatures to check some of our expressions. For this matching the natural approach is a hot hadron gas [43] . Conversely, it would be possible to extend our techniques to the case of isospin chemical potential, that is physically uninteresting but amenable to numerical simulations as the fermion determinant is positive in this case (of course there is no true parity violating phase in this case[12]).

We have seen that SPB appears to be a generic and rather robust possibility offered by QCD at finite density and temperature. This conclusion has been reached by making sure that the low-energy modelling of QCD is as accurate as it is reasonable to expect. We have seen that as a bonus we get a rather good description of several aspects of nuclear physics; in particular a good description of the physics associated to the condensation transition where nuclear matter becomes the preferred solution. The model is rich enough to provide the relevant characteristics while avoiding some undesirable properties of simpler models, such as the chiral collapse. Other nuclear properties such as (in)compressibilities are well described too.

We have presented our results trying to avoid as much as possible specific numerical values for the different quantities and parameters. Not only is this procedure more general but also the logical connections are better outlined. The known restrictions and meson properties of QCD are usually included as inequalities. The main conclusion of our studies is that spontaneous parity breaking may be a rather generic phenomenon at finite density.

XI. ACKNOWLEDGMENTS

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